Advances and Applications in Fluid Mechanics
© 2022 Pushpa Publishing House, Prayagraj, India
http://www.pphmj.com
http://dx.doi.org/10.17654/0973468622003
Volume 28, 2022, Pages 27-40 **P-ISSN: 0973-4686**

MAGNETOHYDRODYNAMIC FLOW THROUGH A POROUS MEDIUM BOUNDED BY AN OSCILLATING POROUS PLATE

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Abstract

A solution is obtained in closed form for the flow of an incompressible viscous fluid of small electrical conductivity in a permeable medium close to an infinite permeable flat plate that oscillates in presence of a transverse magnetic field of uniform strength and stationary relative to the fluid. The graphical portrayals show that the permeable medium lessens the velocity profiles.

Received: April 7, 2022; Accepted: May 27, 2022

2020 Mathematics Subject Classification: 76W05, 76S05.

Keywords and phrases: incompressible fluid, porous medium, transverse magnetic field.

How to cite this article: A. Neelima and V. Omeshwar Reddy, Magnetohydrodynamic flow through a porous medium bounded by an oscillating porous plate, Advances and Applications in Fluid Mechanics 28 (2022), 27-40. http://dx.doi.org/10.17654/0973468622003

Published Online: June 6, 2022

I. Introduction

A permeable medium usually comprises of huge number of interconnected pores each one of which is immersed with the fluid. However, the precise form of the structure is exceptionally intricate and differs from one medium to another medium. A permeable medium can be considered as statistical aggregate of huge number of solid particles containing several capillaries such as porous rock.

Flow through a permeable medium has been of significant interest in recent years especially among geophysical fluid dynamicists. An example in a geophysical context is the recuperation of crude oil from the pores of reservoir rocks. The study of fluid flow through permeable media has most prominent importance in the present century because of their occurrence in nature and newly emerging fields such as petroleum industry and its wide applications in physiology.

Flows through permeable media are a very much common in nature and therefore, the investigation of flows through permeable media has become to guideline premium in numerous engineering and scientific applications. This sort of flows have shown their incredible significance in petroleum engineering in studying the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and water cleansing cycles. Further, to concentrate on the underground water assets, leakage of water in riverbeds and so on one necessities to explore the flows of fluids through permeable media. The permeable medium is in fact a non-homogeneous medium, yet for examination, it very well might be feasible to supplant it with a homogeneous fluid which has dynamical properties equivalent to those of non-homogeneous continuum. Hence, one can concentrate on the flow of a theoretical homogeneous fluid under the action of the properly averaged forces. Subsequently, a convoluted issue of the flow through a permeable medium lessens the flow problem of a homogeneous fluid with some resistance.

The phenomenon of oscillatory flow through a permeable medium has been the object of extensive research due to its possible applications in numerous branches of science and technology. Such unsteady oscillatory flows play an imperative role in chemical engineering, turbo machinery and aerospace technology. In the light of these facts, Rossow [3] investigated on the flow of electrically conducting fluids over a flat plate in presence of a transverse magnetic field. Pande et al. [8] studied unsteady hydro magnetic thermal boundary layer flow. Pande et al. [9] investigated the unsteady hydro magnetic thermal boundary layer flow with no heat transfer. Pande et al. [10] analyzed hydro magnetic flow near an oscillating porous limiting surface. Raptis [13] investigated the problem of unsteady flow through a porous medium bounded by an infinite porous plate subjected to a constant suction and variable temperature. Further, Raptis and Perdikis [14] have presented the problem of free convective flow through a porous medium bounded by a vertical permeable plate with constant suction, when the free stream velocity oscillates in time about a constant mean value. Raptis [15] studied the flow through porous medium in the presence of magnetic field. Takhar et al. [16] investigated the MHD asymmetric flow past a semi-infinite moving plate. Watanabe [17] studied the effect of uniform suction or injection on a magnetohydrodynamic boundary layer flow along a flat plate with pressure gradient. Singh and Dikshit [18] studied the problem on hydro magnetic flow past a continuously moving semi-infinite plate at large suction. Lawrence and Rao [19] investigated the magnetohydrodynamic flow past a semi infinite moving plate. Choudhury and Das [20] studied the MHD boundary layer flows of non-Newtonian fluid past a flat plate. Ahmed and Ahmed [21] presented a problem on two-dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium. Siddique and Mirza [22] studied magnetohydrodynamic free convection flows of a viscoelastic fluid in porous medium with variable permeability heat source and chemical reaction. Kanaujia and Rajput [23] analyzed the joint effect of chemical reaction and hall current on magnetohydrodynamic flow through permeable medium with heat generation past an impulsively started vertical plate with constant wall temperature and mass diffusion.

Verma and Sing [24] studied the MHD flow in a circular channel filled with a porous medium.

Rayleigh [1] investigated the flow about an infinite flat wall which executes linear harmonic oscillations parallel to itself. Scheidegger [2] investigated the flow through a porous media Yamamoto and Yashida [6] investigated flow through a porous wall with convective acceleration. Ong and Nicholls [4] extended the method to obtain the flow in a magnetic field near an infinite flat wall which oscillates in its own plane. Ahmadi and Manvi [5] have derived a general equation of motion through a porous medium and applied the results obtained to some basic flow problems. Yomamoto and Iwamura [11] investigated the flow with convective acceleration through a porous medium.

It is the extension of the research work of Choudhary [12], who studied the MHD flow near an oscillating porous flat plate.

In this problem, an endeavor has been made to study on the movement of an electrically conducting incompressible viscous fluid through a permeable medium close to an infinite oscillating permeable flat plate in the presence of a transverse magnetic field and a fixed relative to the fluid.

II. Mathematical Formation

Let us consider the flow of an electrically conducting fluid of density ρ and viscosity μ through a porous medium of permeability k, occupying a semi-infinite region of the space bounded by a permeable plate. The porosity of the medium is taken to be equal to unity. This supposition that is substantial just for a profoundly permeable medium, for example, an air filter. A uniform magnetic field H_0 is acting along the y-axis. For problems in aeronautical engineering the magnetic Reynolds number R_σ is usually small.

Under such conditions, the induced magnetic field due to the flow might be ignored contrasted with the applied magnetic field. Since the plate has an infinite in length and uniform suction is acts on it, the physical factors rely just upon *y* and *t*. The pressure *p* in the fluid is thought to be constant.

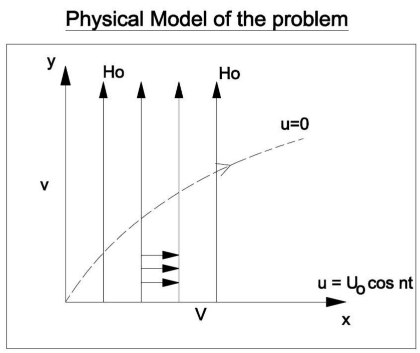

Figure 1

Let u and v be the velocity components in x and y directions correspondingly taken along and perpendicular to the permeable plate. If V addresses the velocity of suction or injection, at the plate, the equation of continuity is

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

with the suction at $y = 0$ and leads to the result $\nu = V$, everywhere.

The boundary layer equation describing the flow of an incompressible viscous electrically conducting fluid through a porous medium (assumed highly porous) is

$$\rho\left[\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y}\right] = \mu\left[\frac{\partial^2 u}{\partial y^2}\right] - \frac{\mu u}{K} - \sigma\mu_e^2 H_0^2 u, \tag{2}$$

where σ is the electrical conductivity and μ_e the magnetic permeability. As the plate executes linear harmonic oscillations in its own plane, the boundary conditions are

$$\left.\begin{aligned} y &= 0, \quad u = U_0 \cos nt, \\ y &\to \infty, \quad u \to 0, \end{aligned}\right\}. \tag{3}$$

Introducing the variables and dropping*,

$$t = nt^*, \quad \eta = y\sqrt{(n/\nu)}, \quad \lambda = V/\sqrt{(n\nu)}, \quad m = \sigma\mu_e^2 H_0^2/\rho,$$

$$M_1 = m/n = R_m^2, \quad K = nk/\nu, \tag{4}$$

we have from equation (1)

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial \eta} + \left[M_1 + \frac{1}{K}\right]u = \frac{\partial^2 u}{\partial \eta^2}, \tag{5}$$

with the condition

$$\begin{aligned} \eta &= 0, \quad u = U_0 \cos t, \\ \eta &\to \infty, \quad u \to 0. \end{aligned} \tag{6}$$

Let the solution be assumed in the form

$$u = F(\eta)e^{-i(t - A\eta)}. \tag{7}$$

Substituting for u in (5) and equating the real and imaginary parts separately to zero, we get

$$F'' - \lambda F' - \left[A^2 + M_1 + \frac{1}{K}\right]F = 0, \tag{8}$$

$$2AF' - [A\lambda - 1]F = 0, \tag{9}$$

where dash denotes differentiation with respect to η.

The solution of the differential equation (9) is

$$F(\eta) = Be^{\frac{(A\lambda - 1)\eta}{2A}}, \tag{10}$$

where B is the constant of integration.

Substituting for F into (8), we have

$$4A^4 + \left[\lambda^2 + 4M_1 + \frac{4}{K}\right]A^2 - 1 = 0, \tag{11}$$

since A^2 is to remain positive, hence

$$A^2 = \frac{\sqrt{\left(\lambda^2 + 4M_1 + \frac{4}{K}\right)^2 + 16} - \left[\lambda^2 + 4M_1 + \frac{4}{K}\right]}{8},$$

and the solution is

$$u = Be^{\frac{\eta\left(\lambda - \frac{1}{A}\right)}{2}} e^{-i(t - A\eta)}. \tag{12}$$

The first of the boundary condition (6) gives $B = U_0$. Hence,

$$u = U_0 e^{\frac{\eta\left(\lambda - \frac{1}{A}\right)}{2}} e^{-i(t - A\eta)}. \tag{13}$$

The real part of (13) is

$$u_1 = U_0 e^{\frac{\eta\left(\lambda - \frac{1}{A}\right)}{2}} \cos(t - A\eta), \tag{14}$$

and the imaginary part of (13) is

$$u_2 = U_0 e^{\frac{\eta\left(\lambda - \frac{1}{A}\right)}{2}} \sin(t - A\eta). \tag{15}$$

III. Results and Discussions

The velocity distribution for the flow of a viscous incompressible fluid electrical conductivity in a porous medium near an infinite oscillating porous flat plate in the presence of a transverse magnetic field fixed relative to the fluid given by

$$\begin{aligned} u = U_0 \exp[(\eta/2)[\lambda - (2\sqrt{2})/\{\sqrt{[(\lambda^2 + 4M_1 + 4/K)^2 + 16]} \\ - (\lambda^2 + 4M_1 + 4/K)\}^{1/2}]] \\ \times \cos[t - (\eta/2^{3/2})\{\sqrt{[(\lambda^2 + 4M_1 + 4/K)^2 + 16]} \\ - (\lambda^2 + 4M_1 + 4/K)\}^{1/2}]. \end{aligned} \tag{16}$$

For $K \to \propto$

$$u = U_0 \exp[(\eta/2)[\lambda - (2\sqrt{2})/\{\sqrt{[(\lambda^2 + 4M_1)^2 + 16]}$$

$$- (\lambda^2 + 4M_1)\}^{1/2}]] \times \cos[t - (\eta/2^{3/2})\{\sqrt{[(\lambda^2 + 4M_1)^2 + 16]}$$

$$- (\lambda^2 + 4M_1)\}^{1/2}] \qquad (17)$$

which is the solution obtained by Choudhary [12] for hydro magnetic flow near an oscillating porous flat plate.

For $\lambda = 0,\ K \to \propto$

$$u = U_0 \exp[\,-(\eta/\sqrt{2})\{\sqrt{(M_1^2 + 1)}\}^{1/2}]$$

$$\times \cos[t - (\eta/\sqrt{2})\{\sqrt{(M_1^2 + 1)} - M_1\}^{1/2}] \qquad (18)$$

which is the solution obtained by Ong and Nicholls [4] for hydro magnetic flow near an oscillating solid flat wall.

For $\lambda = 0,\ M_1 = 0,\ K \to \propto$

$$U = U_0 \exp[-(\eta/\sqrt{2})] \times \cos\{t - (\eta/\sqrt{2})\} \qquad (19)$$

which is the solution given by Stokes [7] for the flow of a viscous, incompressible non-conducting fluid near an infinite oscillating solid plate.

Effects of magnetic parameter M_1 and porous medium parameter *K* on magnitude of velocity $|u|$ for $t = 0$:

The magnitude of velocity is for different values of magnetic parameter M_1 and porous medium parameter *K* with $t = 0,$ suction parameter $\lambda = 0$ and $U_0 = 1$ is shown in Figure 2. It is plotted against the variable $\eta.$ It is observed from the figure that as the magnetic parameter increases, the magnitude of the velocity of the flow field decreases. It is evident from the figure that the presence of the transverse magnetic field (M_1) produces a resistive force on the velocity. This force is called the *Lorentz force* acting on the flow velocity, which leads to slow down the motion of electrically conducting fluid.

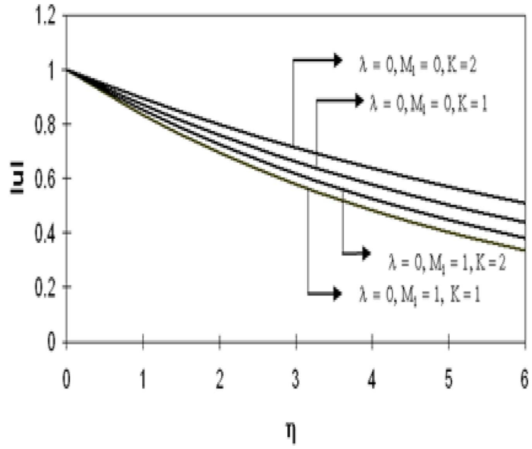

Figure 2. Velocity profile for different values of magnetic parameter M_1 and porous medium parameter K for $t = 0$. The velocity distribution for different values of $M_1 = 0, 1$ and porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1, \lambda = 0$ and $t = 0$.

From Figure 2, it is observed that increase in K leads to an increase in velocity field $|u|$. Hence we conclude that the porous medium reduces the velocity field in the boundary layer.

Effects of magnetic parameter M_1 and porous medium parameter K on magnitude of velocity $|u|$ for $t = \pi/2$:

Figure 3 depicts the variation of $|u| = \sqrt{(u_1^2 + u_2^2)}$ for different values of magnetic parameter M_1 and porous medium parameter K with $t = \pi/2$, suction parameter $\lambda = 0$ and $U_0 = 1$. It is plotted against the variable η. It is observed from the figure that increase in magnetic parameter reduces the velocity profile and increase in porous medium parameter K leads to an increase in velocity field $|u|$.

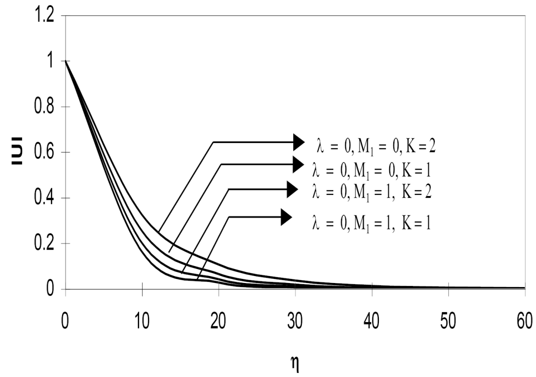

Figure 3. Velocity profile for different values of magnetic parameter M_1 and porous medium parameter K for $t = \pi/2$. The velocity distribution for different values of $M_1 = 0, 1$ and porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1$, $\lambda = 0$ and $t = \pi/2$.

Hence, we conclude from Figures 2 and 3 that magnetic parameter reduces the velocity of the flow field and the porous medium reduces the velocity field in the boundary layer.

Effect of porous medium parameter *K* on phase angle $\arg(u)$ **for** $t = 0$ **and** $M_1 = 0$.

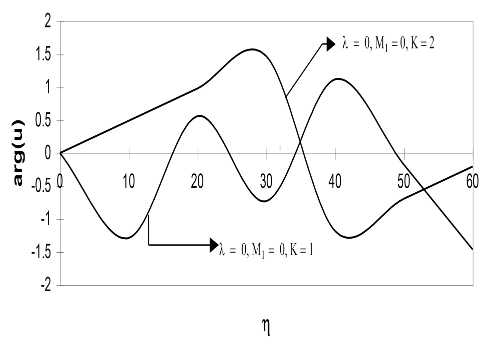

Figure 4. Phase angle $\arg(u)$ for different values of porous medium parameter K for $t = 0$ and $M_1 = 0$. The effect of phase angle for different values of porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1$, $\lambda = 0$, $M_1 = 0$ and $t = 0$.

Figure 4 illustrates the variation of the phase angle of the velocity profile with respect to η for $t = 0$ at the suction parameter $\lambda = 0$ and magnetic parameter $M_1 = 0.$ Since the plate executes linear harmonic oscillations in its own plane, as expected the phase angle varies between -1.5 and 1.5 for $K = 1.$

Effect of porous medium parameter *K* on phase angle $\arg(u)$ **for** $t = \pi/2$ **and** $M_1 = 0$:

Figure 5 shows the variation of the phase angle of the velocity profile with respect to η for $t = \pi/2$ at the suction parameter $\lambda = 0,$ magnetic parameter $M_1 = 0$ and $U_0 = 1.$ Since the plate executes linear harmonic oscillations in its own plane, as expected the phase angle varies between -1.5 and 1.5 for $K = 1.$

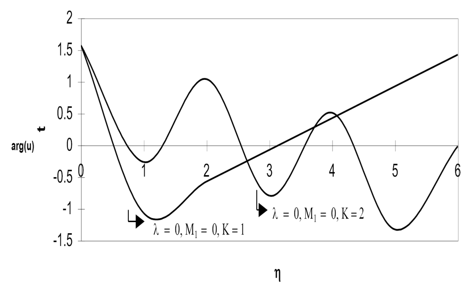

Figure 5. Phase angle $\arg(u)$ for different values of porous medium parameter K for $t = \pi/2$ and $M_1 = 0.$ The effect of phase angle for different values of porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1, \lambda = 0, M_1 = 0$ and $t = \pi/2.$

Effect of porous medium parameter *K* on phase angle $\arg(u)$ **for** $t = 0$ **when** $M_1 = 1$:

Figure 6 shows the variation of $\arg(u)$ for the phase angle of the velocity profile with respect to η for $t = 0,$ for $K = 1$ and $K = 2$ at the suction parameter $\lambda = 0,$ magnetic Reynolds number $M_1 = 1$ and $U_0 = 1.$

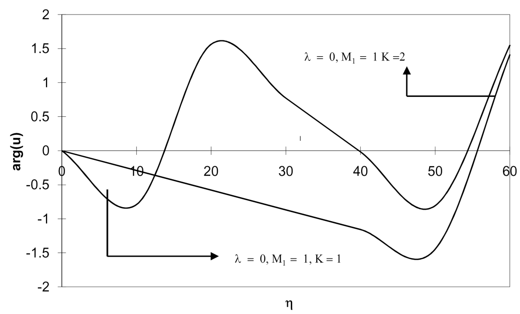

Figure 6. Phase angle $\arg(u)$ for different values of porous medium parameter *K* when $t = 0$ and $M_1 = 1.$ The effect of phase angle $\arg(u)$ for different values of porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1, \lambda = 0,$ $M_1 = 0$ and $t = 0.$

Effect of porous medium parameter *K* on phase angle $\arg(u)$ **for** $t = \pi/2$ **when** $M_1 = 1$:

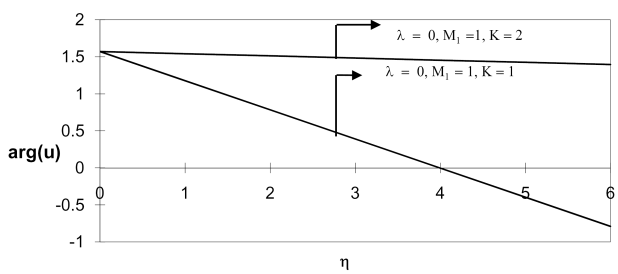

Figure 7. Phase angle $\arg(u)$ for different values of porous medium parameter *K* when $t = \pi/2$ and $M_1 = 1.$ The effect of phase angle $\arg(u)$ for different values of porous medium parameter $K = 1$ and 2 is plotted against the variable η by considering the parameter values as $U_0 = 1,$ $\lambda = 0,$ $M_1 = 1$ and $t = \pi/2.$

Figure 7 gives the variation of $\arg(u)$ for the phase angle of the velocity profile with respect to η for $t = \pi/2,$ for $K = 1$ and $K = 2,$ respectively, at the suction parameter $\lambda = 0$, magnetic Reynolds number $M_1 = 1$ and $U_0 = 1$.

References

[1] Lord Rayleigh, On the motion of solid bodies through viscous liquid, Phil. Mag. 21 (1911), 697-711.

[2] A. E. Scheidegger, The Physics of Flow Through a Porous Media, Univ. Toronoto Pr., 1957.

[3] V. J. Rossow, On the Flow of Electrically Conducting Fluids over a Flat Plate in the Presence of a Transverse Magnetic Field, Nasa Report. 1958, pp. 489-508.

[4] R. S. Ong and J. A. Nicholls, On the flow of a hydromagnetic fluid near an oscillating flat plate, J. Aero - Space Sci. 26 (1959), 313-314.

[5] G. Ahmadi and R. Manvi, Equation of motion for viscous flow through a rigid porous medium, Indian J. Tech. 9 (1971), 441-444.

[6] K. Yamamoto and Z. Yashida, Flow through a porous wall with convective acceleration, J. Phys. Soc. Japan 37 (1974), 774-779.

[7] Shih-I. Pai, Magenetogasdynamics and Plasma Dynamics, Springer-Verlag in Vienna, Austria, 1961, p. 1-72.

[8] G. C. Pande, N. G. Kafousias and C. L. Goudas, Unsteady hydro magnetic thermal boundary layer flow, Astrophys. Space Science 45 (1976), 397-410.

[9] G. C. Pande, N. G. Kafousias and C. L. Goudas, Unsteady hydromagnetic thermal boundary layer flow II, no heat transfer, Astrophys. Space Science 50 (1977), 141-152.

[10] G. C. Pande, N. G. Kafousias, G. A. Georgantopoulos and C. L. Goudas, Hydromagnetic flow near an oscillating porous limiting surface, Astrophys. Space Science 51 (1977), 125-134.

[11] K. Yamamoto and N. Iwamura, Flow with convective acceleration through a porous medium, J. Engg. Math. 10 (1976), 41-54.

[12] R. C. Choudhary, Hydromagnetic flow near an oscillating porous flat plate, Indian Acad. Sci. A 86 (1977), 531-535.

[13] A. Rapits, Unsteady free convective flow through a porous medium, International Journal of Engineering Science 21 (1983), 345-348.

[14] A. Raptis and C. P. Perdikis, Oscillatory flow through a porous medium by the presence of free convective flow, Int. J. Engng. Sci. 23 (1985), 51-55.

[15] A. Raptis, Flow through porous medium in the presence of magnetic field, Int. J. Energy Res. 10 (1986), 97-100.

[16] H. S. Takhar, A. A. Raptis and C. P. Perdikis, MHD asymmetric flow past a semi-infinite moving plate, Acta Mechanica 65(1-4) (1987), 287-290.

[17] T. Watanabe, Effect of uniform suction or injection on a magneto-hydrodynamic boundary layer flow along a flat plate with pressure gradient, Acta Mechanica 73 (1988), 33-44.

[18] A. K. Singh and C. K. Dikshit, Hydromagnetic flow past a continuously moving semi-infinite plate at large suction, Astrophys. Space Sci. 148 (1988), 249-256.

[19] P. S. Lawrence and B. N. Rao, Magneto hydrodynamic flow past a semi infinite moving plate, Acta Mech. 117 (1996), 159-164.

[20] R. Choudhury and A. Das, Magneto-hydrodynamic boundary layer flows of non-Newtonian fluid past a flat plate, Indian J. Pure Appl. Math. 31(11) (2000), 1429-1441.

[21] S. Ahmed and N. Ahmed, Two dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium, Indian J. Pure Appl. Math. 35(12) (2004), 1309-1319.

[22] Imran Siddique and Itrat Abbas Mirza, Magneto-hydrodynamic free convection flows of a viscoelastic fluid in porous medium with variable permeability heat source and chemical reaction, Results in Physics 7 (2017), 3928-3937.

[23] Neetu Kanaujia and Uday Singh Rajput, Combined effect of hall current and chemical reaction on MHD flow through porous medium with heat generation past an impulsively started vertical plate with constant wall temperature and mass diffusion, Summer and Autumn 9(1) (2019), 121-132.

[24] Vineet Kumar Verma and Sanjeeva Kumar Sing, Magnetohydrodynamic flow in a circular channel filled with a porous medium, Journal of Porous Media 18(9) (2015), 923-928.