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Bipolar sum distance in neutrosophic graphs

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Abstract

Distance is one of the most branches in graph theory. In this manuscript authors are derived the bipolar sum distance in neutrosophic graphs. Here we are using NG with weighted edges derived sum of the distance based on fixed weights. In this present article we derived properties of metric and some terminology on NGs.

Keywords

Neutrosophic graph, weight of edges, strong sum distance.

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1. Introduction

1.1 Notations

1. Neutrosophic set (NS)
2. Bipolar Neutrosophic set (BNS)
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4. Single Valued Neutrosophic Set (SVNS).
5. Single Valued Neutrosophic Graph (SVNG).
6. Intutionistic Fuzzy Garph(IFG)
7. Bipolar Single Valued Neutrosophic Graph (BSVNG).

Tom and Sunitha successfully introduced strong sum distance concept in 2015. In this logic we use the membership functions T, I, F with respect to truth indeterminacy and falsity. The values of T, I and F are lies in the interval $]-0,1^{+}[$, where $0 \leq T+I+F \leq 3$. Beneficial to concern practically actual world problems H. Wang et. al. [2] establish the idea of a SVNS by defining T, I and F are belongs to $[0,1]$ which are subclass of the $NS]-0,1^{+}[$.

Neutrosophy is deducement of theory of fuzzy set [3], intuitionistic fuzzy sets [4]. S. Broumi et.al. [7] established the concept of SVNG and proved that deducement of fuzzy graph and IFG. I. Deli et.al. [5] determine the theory of BNS as extension of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set. By using the concept of bipolar neutrosophic set and graph theory S. Broumi et.al. [6]. Introduced the BSVNG, strong bipolar SVNG and other concepts. Ch. Shashi Kumar et.al [15, 16, 17] discussed about BSVNG interior and boundary vertices with distance. The same is extended about bipolar SVNG and neutrosophic detour distance between vertices of the graph by V. Venkateswara rao et.al [7, 8, 9, 10, 11, 12, 13, 14].

2. Preliminaries

Definition 2.1. *A NG is $=(V,E)$, where edge set is subset of Cartesian product of the vertex set if*

(i) for some functions $\rho^{T}: V \rightarrow [0,1], \rho^{F}: V \rightarrow [0,1]$ and $\rho^{I}: V \rightarrow [0,1]$ such that $0 \leq \rho^{T}(v_i)+\rho^{F}(v_i)+\rho^{I}(v_i) \leq 3$ for all $v_i \in V (i=1,2,3,\ldots,n)$ where $(v_i), \rho^{F}(v_i), \rho^{I}$ (v_i) these values are lies between 0 to 1.

(ii) for some functions $\mu^{T}: E \rightarrow [0,1], \mu^{F}: E \rightarrow [0,1]$ and $\mu^{I}: E \rightarrow [0,1]$ such that
$(v_i, v_j) \leq \min[(vi),(vj)]$
$\mu^{F}(v_i, v_j) \geq \max$

$\left[\rho^F(v_i), \rho^F(v_j)\right]$
$\mu^I(v_i, v_j) \geq \max\left[\rho^I(v_i), \rho^I(v_j)\right]$
and $0 \leq \mu^T(v_i, v_j) + \mu^F(v_i, v_j) + \mu^I(v_i, v_j) \leq 3$ for all $(v_i, v_j) \in E$
where $\mu^T(v_i, v_j), \mu^F(v_i, v_j), \mu^I(v_i, v_j)$ the values are lies between 0 to 1 .

Definition 2.2. *Consider a function $\omega : [0,1]\times[0,1]\times[0,1] \to [0,1]$ defined by $\omega_{ij}(t,i,f) = w_1 t(1-f) + w_2 i$ where $t, i, f, w_1, w_2 \in [0,1]$. The edge (vi, vj) weight in a NG is liesin [0 1] derived from the image of the function ω for analogous values $\left(T^E(v_i,v_j), F^E(v_i,v_j), IE(v_i,v_j)\right)$ of the edge and it is denoted by ω_{ij}.*

Definition 2.3 (Bipolar neutrosophic graphs)**.** *A bipolar neutrosophic graph (BN-graph) with N_V is explained by to be a two of a kind (P,Q) everywhere $[0,1]$ indicate the interval B.*

The functions $T_P : N_V \to B, I_P : N_V \to B$ and $F_P : N_V \to B$ and $0 \leq T_P + I_P + F_P \leq 3$ for all vertices in N_V. Further,

The functions $T_Q : N_V \times N_V \to B, I_Q : N_V \times N_V \to B$ and $F_Q : N_V \times N_V \to B$ are explained by $T_Q(a_i,a_j) \leq \min[T_Q(a_i), T_Q(a_j)], I_Q(a_i,a_j) \geq \max[I_Q(a_i), I_Q(a_j)]$ and $F_Q(a_i,a_j) \geq \max[F_Q(a_i), F_Q(a_j)]$ with the condition

$$0 \leq T_B(a_i,a_j) + I_B(a_i,a_j) + F_B(a_i,a_j) \leq 3$$

for all $(a_i,a_j) \in E$.

Definition 2.4. *Consider a function $\omega : [-1,0]\times[-1,0]\times[-1,0]\times[0,1]\times[0,1]\times[0,1] \to [0,1]$ explained by $BN\omega_{ij}(T;I,F;T^+,I^+,F^+) = \left[w_1\left(1+T^-\right)\left(1+F^-\right) + w_2(1+I)\right] + \left[w_1\left(T^+\right)\left(1-F^+\right) + w_2\left(I^+\right)\right]$ where T^-, I, F^- are the number $\in [-1,0]$. $T^+, I^+, F^+, w_1, w_2 \in [0,1]$. An edge $(v_{i,j})$ weight in a NG is lies in $[01]$.*

Definition 2.5. *Let $P : u_0, u_1, u_m$ be arbitrary bipolar path in a NG $G = (V,E)$. Then the length of the bipolar path P is the sum of bipolar weighted edges of the bipolar path P in $G = (V,E)$. $BL_N(p) = \Sigma BN\omega_{ij} 0 \leq i < n_i < j$, Where ω_{ij} is the bipolar weighted edge $(u_i - u_j)$.*

Definition 2.6. *Let $G = (V,E)$ be a bipolar NG and P be the group of all bipolar paths of any two vertices u and v i.e. $P = \{p_i, i = 1,2,3,\ldots\ldots,n\}$. Then the bipolar weighted distance between any two vertices uandv is indicated by $Bd_N(u,v)$ and is explained by*

$$Bd_N(u,v) = \min\{BL_N(p_i) : p_i \in P, i = 1,2,3,\ldots\ldots,n\}$$

Where $B(p_i)$ is the bipolar length of p_i.

Definition 2.7. *The bipolar eccentricity $B(u)$ of a point, u is the bipolar weighted distance taken away u to any other point in NG G. Thus $Be_N(u) = \max\{Bd_N(u,v) : \forall v \in V\}$.*

Definition 2.8. *The bipolar radius $Br_N(G)$ of a bipolar NG, G is the least encompassed by entire bipolar eccentricity of vertices. Thus. $B(G) = \min\{Be_N(u) : \forall u \in G\}$.*

Definition 2.9. *The bipolar diameter $Bd_N(G)$ of a bipolar NG, G is the greatest encompassed by entire bipolar eccentricity of vertices. Thus, $B(G) = \max\{\mathrm{B}\,e_N(u) : \forall u \in G\}$.*

Definition 2.10. *A bipolar NG of a vertex is said to be central vertex if eccentricity is equal to the radius with respect to bipolar NG. Thus, central vertexu, $B(u) = \mathrm{Br}_N(G)$.*

Definition 2.11. *A bipolar NG of a vertex is said to be peripheral vertex if its bipolar eccentricity is equal to their diameter with respect to bipolar NG. Thus, peripheral node u, $B(u) = Bd_N(G)$.*

3. Main Results

Theorem 3.1. *Let $G = (V,E)$ be any bipolar NG and $B(u,v)$ be sum distance in bipolar weighted in any two vertices u and v. Then $\forall u, v \in V$.*

(i) $Bd_N(u,v) \geq 0$

(ii) $Bd_N(u,v) = 0$ *if and only if* $u = v$

(iii) $Bd_N(u,v) = Bd_N(v,u)$

(iv) $B(u,v) \leq Bd_N(u,w) + Bd_N(w,v)$

Proof. (i) From the explanation, satisfies the condition B $d_N(u,v) \geq 0$.

(ii) It clears from the definition that $Bd_N(u,v) = 0$ if and only if $u = v$.

(iii) $Bd_N(u,v)$ indicates the bipolar strong sum distance any two vertices u and v. Then for some bipolar path of length is least encompassed by entire bipolar path any two vertices u and v. Hence the bipolar length is similar from v to u. So $Bd_N(u,v) = Bd_N(v,u)$.

(iv) Let p be a bipolar path $u - w$ such that $BL_N(p) = Bd_N(u,w)$ and q be a bipolar path $w - v$ such that $L_N(q) = Bd_N(w,v)$. Then $u - v$ is a walk and it is a bipolar strong path of length is maximum $Bd_N(u,w) + Bd_N(w,v)$. Thus $B(u,v) \leq Bd_N(u,w) + Bd_N(w,v)$.

□

Theorem 3.2. *Let $G = (V,E)$ be a connected bipolar NG and u, v be any two vertices of G. Then $|Be_N(u) - Be_N(v)| \leq Bd_N(u,v)$.*

Proof. Let $u, v \in G$ be two vertices such that $B(u) \geq Be_N(v)$ and $x \in G$ be a vertex such that $\mathrm{B}\,e_N(u) = Bd_N(u,x)$. Then $B(u,x) \leq Bd_N(u,v) + Bd_N(v,x)$.
By theorem: 1 (iv). Also $Bd_N(v,x) \leq \mathrm{B}\,e_N(v)$. Thus $Be_N(u) = Bd_N(u,x) \leq Bd_N(u,v) + \mathrm{Be}_N(v)$.
Which gives that, $0 \leq B(u) - \mathrm{B}\,e_N(v) \leq Bd_N(u,v)$. Correspondingly, if consider $Be_{\ N}(u) \leq Be_N(v)$, we obtained,

$$-Bd_N(u,v) \leq Be_N(u) - Be_N(v).$$

Thus, $|\ Be_N(u) - \mathrm{B}\,e_N(v)\ | \leq Bd_N(u,v)$. □

Theorem 3.3. *Let $G=(V,E)$ be a connected bipolar NG with* $\mathrm{B}\,r_N(G)$ *and* $Bd_N(G)$ *be the bipolar radius and bipolar diameter in combination, then* $Br_N(G) \leq Bd_N(G) \leq 2\,Br_N(G)$.

Proof. From the definition, it follows that $Br_N(G) \leq Bd_N(G)$. Let $u_3 v \in V$ such that u be central vertex. i.e. $B(u) = \mathrm{Br}_N(G)$ and v,w be peripheral vertex i.e. $\mathrm{Be}_N(v) = \mathrm{Be}_N(w) = \mathrm{B}\,d_N(G)$.

Now $B(v,w) \leq Bd_N(v,u) + Bd_N(u,w)$ from theorem: 1 (iv), which gives $Bd_N(G) \leq \mathrm{Br}_N(G) + \mathrm{Br}_N(G) = 2\,Br_N(G)$. Thus $Bd_N(G) \leq 2\,Br_N(G)$. Therefore, $\mathrm{Br}_N(G) \leq Bd_N(G) \leq \mathrm{B}\,r_N(G)$. □

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