

## SINGLE VALUED NEUTROSOPHIC DETOUR DISTANCE

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### Abstract

In the present article, we deduce a characterization of SVN detour eccentric vertex. The concepts of SVN graph are examined. Also we obtained some definitions SVN on a vertex like SVN detour eccentric vertex, SVN detour radius, SVN detour diameter, SVN detour centered and SVN detour periphery. We derive some important results based on these SVN detour radius, diameter, center and periphery.

**Keywords** --- Detour distance, SVN detour eccentric, SVN detour distance

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### INTRODUCTION

The Neutrosophic sets launch by Smarandache [10, 11] are a great exact implement for the situation uncertainty in the real world. These uncertainty idea comes from the theories of fuzzy sets [5], intuitionistic fuzzy sets [2, 4] and interval valued intuitionistic fuzzy sets [3]. The representation of the neutrosophic sets are truth, indeterminacy and falsity value. These T, I, F values belongs to standard or nonstandard unit interval denoted by  $]-0, 1+[$  [6,9].

The idea of subclass of the NS and SVNS introduced by Wang et al. [12]. The idea of SVNS initiation by intuitionistic fuzzy sets [1,7], in this the functions Truth, Indeterminacy, Falsity are not dependent and these values are present within  $[0, 1]$  [8]. Neutrosophic theory is widely expands in all fields especially authors discoursed about topology with respect to neutrosophic [13].

Graph theory has at this time turn into a most important branch of mathematics. It is the division of combinatory. The Graph is a extensively important to analyze combinatorial complication in dissimilar areas in mathematics, optimization and computer science. Mainly significant object is well-known. The uncertainty on the subject of vertice and edges or both representation to be a fuzzy graph.

In this manuscript, discuss about SVN graphs and neutrosophic detour distance between two vertices of the graph based on this define SVN eccentricity, radius, diameter, center and periphery with respect to detour distance. Also find some important results on these topics.

### PRELIMINARIES

#### Explanation 2.1 SVN sets

A SVN set is explained as the membership functions represented as a triplet set in  $W$  is denoted by  $\{ \langle w, T, I, F \rangle : w \in W \}$ , these functions are mapping from  $W$  to  $[0, 1]$ . Where  $T$  denote truth membership,  $I$  denote indeterminate value and  $F$  denote false value of  $W$ .

**Example.** Let  $W = \{ w_1, w_2, w_3 \}$

$$A = \{ \langle w_1, 0.3, 0.2, 0.7 \rangle, \langle w_2, 0.5, 0.3, 0.1 \rangle, \langle w_3, 0.8, 0.05, 0.4 \rangle \}$$

is a SVN set in  $W$

#### Explanation 2.2 SVN relation on $W$

Let  $W$  be a non-empty set. Then we call mapping

$$A = (W, T, I, F)$$

$F(w_1, w_2) : W \times W \rightarrow [0, 1] \times [0, 1]$  is a SVN

relation on  $W$  such that

$$T_A(w_1, w_2) \in [0, 1], I_A(w_1, w_2) \in [0, 1], F_A(w_1, w_2) \in [0, 1].$$

#### Explanation 2.3.

Let  $A = (T_A, I_A, F_A)$  and  $B = (T_B, I_B, F_B)$  be a SVN graphs on a set  $W$ . If  $B$  is a SVN relation on  $A$ , then

$$T_B(w_1, w_2) \leq \min(T_A(w_1), T_A(w_2))$$

$$I_B(w_1, w_2) \geq \max(I_A(w_1), I_A(w_2))$$

$$F_B(w_1, w_2) \geq \max(F_A(w_1), F_A(w_2)) \quad \text{for all}$$

$$w_1, w_2 \in W$$

#### Explanation 2.4.

The symmetric property defined on SVN relation  $B$  on  $W$  is explained by

$$\begin{aligned} T_B(w_1, w_2) &= T_B(w_2, w_1) \\ I_B(w_1, w_2) &= I_B(w_2, w_1) \\ F_B(w_1, w_2) &= F_B(w_2, w_1) \end{aligned}$$

**Explanation 2.5. SVN graph**

The new graph in SVN is denoted by  $G^* = (V, E)$  is a pair  $G = (A, B)$ , where  $A = (T_A, I_A, F_A)$  is a SVN in  $V$  and  $B = (T_B, I_B, F_B)$  is SVN in  $V^2$  defined as

$$\begin{aligned} T_B(w_1, w_2) &\leq \min(T_A(w_1), T_A(w_2)) \\ I_B(w_1, w_2) &\geq \max(I_A(w_1), I_A(w_2)) \\ F_B(w_1, w_2) &\geq \max(F_A(w_1), F_A(w_2)) \end{aligned} \quad \text{for all } w_1, w_2 \in V$$

SVNG of an edge denoted by  $w_1 w_2 \in V^2$

**Explanation 2.6.**

Let  $G = (A, B)$  be a SVNG and  $x, y \in V$

$$(T_B(r, s), I_B(r, s), F_B(r, s))^k = \begin{cases} \sup \{T_B(r, w_1) \wedge T_B(w_1, w_2) \wedge \dots \wedge T_B(w_{k-1}, s)\}, \\ \inf \{I_B(r, w_1) \vee I_B(w_1, w_2) \vee \dots \vee I_B(w_{k-1}, s)\}, \\ \inf \{F_B(r, w_1) \vee F_B(w_1, w_2) \vee \dots \vee F_B(w_{k-1}, s)\} \end{cases}$$

$(T_B(r, s), I_B(r, s), F_B(r, s))^\infty$  is said to be the strength of connectedness between two vertices  $x$  and  $y$  in  $G$ , where

$$(T_B(r, s), I_B(r, s), F_B(r, s))^\infty = \left( \sup_{k \in \mathbb{N}} \{T_B(r, s)\}, \inf_{k \in \mathbb{N}} \{I_B(r, s)\}, \inf_{k \in \mathbb{N}} \{F_B(r, s)\} \right)$$

if  $(T_B(r, s) \geq (T_B(r, s))^\infty, I_B(r, s) \leq (I_B(r, s))^\infty, F_B(r, s) \leq (F_B(r, s))^\infty)$  then the arc  $rs$  in  $G$  is called a strong arc.

A path  $x - y$  is strong path if all arcs on the path are strong.

**SVN DETOUR DISTANCE**

**Explanation 3.1**

SVN detour distance is defined as the length of,  $x - y$  strong path between  $x$  and  $y$  if there is no other strong path longer than  $P$  between  $x$  and  $y$  and we denote this by  $S.N.D(x, y)$ . Any  $x - y$  strong path whose length is  $S.N.D(x, y)$  is called a  $x - y$  SVN detour path.

**SVN DETOUR PERIPHERY ( $Per_{S.N.D}(G)$ ) AND SVN DETOUR ECCENTRIC SUB GRAPH ( $ECC_{S.N.D}(G)$ )**

**Theorem 4.1**

A SVN graph  $G$  is a SVN detour self centered if and only if every node of  $G$  is a SVN detour eccentric.

**Proof.** Suppose  $G$  is a SVN detour self centered SVN graph and let  $b$  be a node in  $G$ . Let  $a \in b_{S.N.D}^*$ . So

$e_{S.N.D}(b) = S.N.D(a, b)$ . Since  $G$  is a SVN detour self centered SVN graph,  $e_{S.N.D}(a) = e_{S.N.D}(b) = S.N.D(a, b)$  and this

A path  $P : x = w_0, w_1, w_2, \dots, w_{k-1}, w_k = y$  in  $G$  is sequence of distinct vertices such that  $(T_B(w_{i-1}, w_i) > 0, I_B(w_{i-1}, w_i) > 0, F_B(w_{i-1}, w_i) > 0)$ ,  $i = 1, 2, \dots, k$  and length of the path is  $k$ , where  $x$  is called initial vertex and  $y$  is called terminal vertex in the path.

**Explanation 2.7.**

A SVN graph  $G = (A, B)$  of  $G^* = (V, E)$  is called strong SVN graph if

$$\begin{aligned} T_B(w_i, w_j) &= \min(T_A(w_i), T_A(w_j)) \\ I_B(w_i, w_j) &= \max(I_A(w_i), I_A(w_j)) \\ F_B(w_i, w_j) &= \max(F_A(w_i), F_A(w_j)) \end{aligned} \quad \text{for all } (w_i, w_j) \in E$$

If  $P : r = w_0, w_1, w_2, \dots, w_{k-1}, w_k = s$  be a path of length  $k$  between  $r$  and  $s$  then

$(T_B(r, s), I_B(r, s), F_B(r, s))^k$  is defined as

implies that  $b \in a_{S.N.D}^*$ . Hence  $b$  is a SVN detour eccentric node of  $G$ .

Conversely, let each vertex of  $G$  is a SVN detour eccentric vertex. If possible, let  $G$  be not SVN detour self centered SVN graph. Then  $rad_{S.N.D}(G) \neq diam_{S.N.D}(G)$  and  $\exists$  a node  $r \in G$  such that  $e_{S.N.D}(r) = diam_{S.N.D}(G)$ . Also let  $p \in r_{S.N.D}^*$ . Let  $U$  be a  $r - p$  SVN detour in  $G$ . So there must have a node  $q$  on  $U$  for which the node  $q$  is not a SVN detour eccentric node of  $U$ . Also  $q$  cannot be a SVN detour eccentric node of every other node. Again if  $q$  be a SVN detour eccentric node of a node  $a$  (say), means  $q \in a_{S.N.D}^*$ .

Then there exist an extension of  $a - q$  SVN detour up to  $r$  or up to  $p$ . But this contradicts the facts that  $q \in a_{S.N.D}^*$ . Hence  $rad_{S.N.D}(G) = diam_{S.N.D}(G)$  and  $G$  is a SVN detour self centered SVN graph.

**Theorem 4.2.** If  $G$  is a SVN detour self centered SVN graph, then  $rad_{S.N.D}(G) = diam_{S.N.D}(G) = n - 1$ , here  $n$  is order of the graph  $G$ .

**Proof.** Let  $G$  be a SVN detour self centered SVN graph. If possible, let  $diam_{S.N.D}(G) = l < n - 1$ .

Let  $U_1$  and  $U_2$  be two distinct SVN detour peripheral path. Let  $p \in U_1, q \in U_2$ . So  $\exists$  a strong path between  $p$  and  $q$ , because of connectedness of  $G$ . Then  $\exists$  nodes on  $U_1$  and  $U_2$ , whose eccentricity  $> l$ , but this is impossible, because  $diam_{S.N.D}(G) = l$ . Hence  $U_1$  and  $U_2$  are not distinct. Since  $U_1$  and  $U_2$  are arbitrary, so  $\exists$  node  $r$  in  $G$  such that  $r$  is a common in all SVN detour peripheral paths. So  $e_{S.N.D}(r) < l$ , which is impossible, because  $G$  is a SVN detour self centered. Hence  $diam_{S.N.D}(G) = n - 1 = rad_{S.N.D}(G)$ .

**Theorem 4.3.** For a connected SVN graph  $G$ ,  $Per_{S.N.D}(G) = G$  if and only if the SVN detour eccentricity of each node of  $G$  is  $n-1$ ,  $n =$  number of nodes in  $G$ .

**Proof.** Let  $Per_{S.N.D}(G) = G$ . Then  $e_{S.N.D}(p) = diam_{S.N.D}(G), \forall p \in G$ . So each vertex of  $G$  is a SVN detour periphery vertex of  $G$ . Hence,  $G$  is a self centered SVN graph and  $rad_{S.N.D}(G) = diam_{S.N.D}(G) = n - 1$ . So the SVN detour eccentricity of each node of  $G$  is  $n - 1$ .

Conversely, let the SVN detour eccentricity of each node of  $G$  is  $n - 1$ . So  $rad_{S.N.D}(G) = diam_{S.N.D}(G) = n - 1$ . All nodes of  $G$  are SVN detour peripheral nodes and hence  $Per_{S.N.D}(G) = G$ .

**Theorem 4.4.** For a connected SVN graph  $G$ ,  $Ecc_{S.N.D}(G) = G$  if and only if the SVN detour eccentricity of each node of  $G$  is  $n-1$ ,  $n =$  number of nodes in  $G$ .

**Proof.** Let  $Ecc_{S.N.D}(G) = G$ , So all nodes of  $G$  are SVN detour eccentric node. Therefore  $G$  is self centered SVN graph and  $diam_{S.N.D}(G) = n - 1$ . Hence the SVN detour eccentricity of each node of  $G$  is  $n - 1$ .

Conversely, let the SVN detour eccentricity of each node of  $G$  is  $n - 1$ . So  $rad_{S.N.D}(G) = diam_{S.N.D}(G) = n - 1$ . So all nodes of  $G$  are SVN detour peripheral nodes as well as SVN detour eccentric node. Hence,  $Ecc_{S.N.D}(G) = G$ .

**Theorem 4.5.** In a connected SVN graph  $G$ , a node  $b$  is a SVN detour eccentric node iff  $b$  is a SVN detour peripheral node.

**Proof.** Let  $b$  be a SVN detour eccentric node of  $G$  and let  $b \in a_{S.N.D}^*$ . Let  $x$  and  $y$  be two SVN detour peripheral nodes, then  $S.N.D(x, y) = diam_{S.N.D}(G) = k$  (say). Let  $P_1$  and  $P_2$  be any  $x - y$  and  $a - b$  SVN detour in  $G$  respectively. There arise two cases.

**Case 1:** When  $b$  is not internal node in  $G$  i.e, there is only one node, say  $c$  which is adjacent to  $b$ . So  $c \in P_2$ . Since  $G$  is connected,  $c$  is connected to a node of  $P_1$ , say  $c'$ . So either  $c' \in P_2$  or  $c' \in (P_1 \cap P_2)$ . Thus in any case the path from  $a$  to  $x$  or  $a$  to  $y$  through  $c$  and  $c'$  is longer than  $P_2$ . But it is impossible, since  $b$  is a SVN detour eccentric node of  $a$ . Hence  $e_{S.N.D}(a) = diam_{S.N.D}(G)$  i.e,  $b$  is a SVN detour peripheral node of  $G$ .

**Case 2:** When  $b$  is internal node in  $G$ , then  $\exists$  a connection between  $b$  to  $x$  and  $b$  to  $y$ , because of connectedness of  $G$ . Then  $a - b$  SVN detour can be extend to  $x$  or  $y$ . This is impossible, because  $b$  is a SVN detour eccentric node of  $a$ . Hence  $e_{S.N.D}(a) = diam_{S.N.D}(G)$  i.e,  $b$  is a SVN detour peripheral node of  $G$ .

Conversely, we assume that  $b$  be a SVN detour peripheral node  $G$ . So  $\exists$  a SVN detour peripheral node, say  $a$  (distinct from  $b$ ). Therefore  $b$  is a SVN detour eccentric node of  $a$ .

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