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SERIES SOLUTIONS OF MHD PERISTALTIC TRANSPORT WITH THE IMPACT OF THERMAL RADIATION

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ABSTRACT

This article intends to relate the thermal radiation effect on MHD peristaltic transport of the Eyring-Powell fluid in a channel with compliant walls under slip conditions for velocity, temperature, and concentration. Assumptions of a long wave length and low Reynolds number are considered. The modeled equations are solved by using the perturbation method. The resulting non-linear system is solved for the stream function, velocity, temperature and concentration. The flow quantities are examined for various parameters. Temperature decreases with an increase in the radiation parameter, while the opposite effect is observed for the concentration.

***Keywords**: Thermal radiation; MHD; Peristalsis; Eyring-Powell fluid; Compliant walls .*

I. INTRODUCTION

The mechanism of the pumping fluid in a channel/tube from a region of minor pressure to major one is known as peristalsis. The peristaltic phenomenon is unique in the sense that the fluid is transported by the action of a progressive sinusoidal wave due to area contraction or expansion, which propagates along an extensible tube/channel instead of using a piston. Peristaltic transport of Newtonian and non-Newtonian fluids has been extensively recognized by several authors due to its numerous physiological and engineering applications. In physiology, the peristalsis occurs in swallowing food through esophagus, chyme movement in the gastrointestinal tract, urine transport from kidney to bladder through the ureter, and vasomotion of blood vessels in capillaries and arterioles. The earliest study about the mechanism of peristaltic transport of a viscous fluid was carried out by Latham [1]. Shapiro et al. [2] presented a mathematical model for peristaltic pumping under the assumptions of a long wavelength and low Reynolds number approximation.. Some useful attempts in this direction were described in [3 – 5]. The analysis in all these attempts has been made by employing no-slip boundary conditions and one or more simplified assumptions of a long wavelength, low Reynolds number, small wave number, small amplitude ratio, etc.

Radiation heat transfer plays a major role in cooling of electronics as well as conduction and convection heat transfer. Thermal radiation has many applications such as cooling, furnaces, boilers, piping and solar radiation in

The aim of the present investigation is to study the impacts of the thermal radiation on an MHD peristaltic flow of the Eyring-Powell fluid with wall properties. A mathematical model of the flow, heat and mass transfer for the present problem is constructed with imposing long wave length and low Reynolds number approximations. Series solutions for a small fluid parameter are developed by a regular perturbation approach. Numerical results for the emerging non-dimensional parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number, and Sherwood number are shown graphically and analyzed. Also, the trapping phenomenon is presented for the pertinent parameters.

II. MATHEMATICAL MODEL

The peristaltic transport of an incompressible viscous electrically conducting, radiating and reacting flow of Eyring-Powell fluid filled with a porous medium in a two-dimensional channel of width $2d_1$ is considered. We have chosen

Cartesian coordinates (x, y) with x in the direction of wave propagation and y transverse to it. The flow geometry of the present problem is presented in Fig. 1. The walls of the channel are flexible which are considered as stretched membrane with viscous damping forces. Sinusoidal waves of long wavelength are assumed to travel with speed c along the channel walls. The configuration of the wall surface is given by

$$y = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right] \quad (1)$$

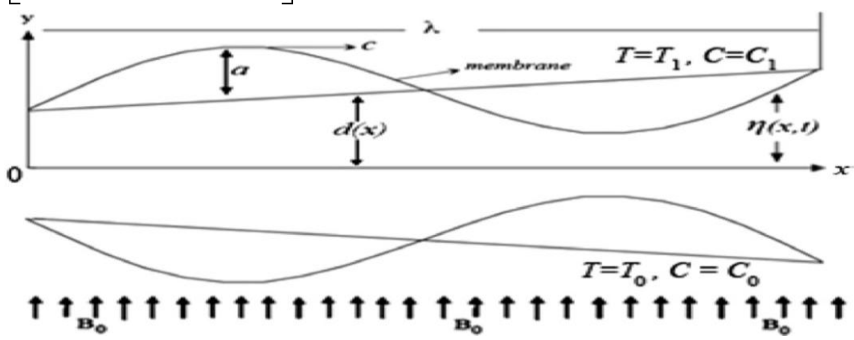

Fig.1. Geometry of the problem

Introducing the stream function $\psi(x,y,t)$

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}. \quad (2)$$

And defining dimensionless quantities

$$x^* = \frac{x}{\lambda},\ y^* = \frac{y}{d_1},\ \psi^* = \frac{\psi}{cd_1},\ t^* = \frac{ct}{\lambda},\ \eta^* = \frac{\eta}{d_1},\ p^* = \frac{pd_1^2}{c\lambda\mu},\ B = \frac{1}{\mu\beta c_1},\ A = \frac{Bc^2}{2d_1^2c_1^2},\ \delta = \frac{d_1}{\lambda},$$

$$\theta = \frac{T-T_o}{T_o},\ \varphi = \frac{C-C_o}{C_o},\ \mathrm{Re} = \frac{\rho cd_1}{\mu},\ M = \sqrt{\frac{\sigma}{\mu}}B_o d_1,\ E_1 = \frac{-\tau d_1^3}{\lambda^3\mu c},\ E_2 = \frac{mcd_1^3}{\lambda^3\mu},\ E_3 = \frac{dd_1^3}{\lambda^2\mu}, \quad (3)$$

$$\beta_i^* = \frac{\beta_i}{d_i}(i=1-3),\ \mathrm{Pr} = \frac{\mu c_p}{k},\ Ec = \frac{c^2}{C_pT_o},\ Sc = \frac{\mu}{D\rho},\ Sr = \frac{\rho DK_TT_o}{\mu T_mC_o}, Rd = \frac{16\sigma^*T_o^3}{3k^*\bar{\mu}_oc_f},\ Br = \mathrm{Pr}\,Ec$$

Using Eq. (2) and quantities (3) into goverening equations and to apply the long wavelength and small Reynolds number approximations, we obtain the following non-dimensional equations (after dropping asterisks)

$$(1+B)\frac{\partial^3\psi}{\partial y^3} - \frac{A}{3}\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^3 - \left(M^2\right)\frac{\partial\psi}{\partial y} = \frac{\partial p}{\partial x}, \quad (4)$$

$$\frac{\partial p}{\partial y} = 0, \quad (5)$$

$$\left(\frac{1}{\mathrm{Pr}} + Rd\right)\frac{\partial^2\theta}{\partial y^2} + Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\left[(1+B) - \frac{A}{3}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\right] = 0, \quad (6)$$

$$\frac{\partial^2\varphi}{\partial y^2} + ScSr\frac{\partial^2\theta}{\partial y^2} = 0. \quad (7)$$

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The corresponding boundary conditions are

$$\frac{\partial}{\partial x}\left(E_1\frac{\partial^2}{\partial x^2}+E_2\frac{\partial^2}{\partial t^2}+E_3\frac{\partial}{\partial t}\right)\eta=(1+B)\frac{\partial^3\psi}{\partial y^3}-\frac{A}{3}\frac{\partial}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2}\right)^3-\left(M^2\right)\frac{\partial\psi}{\partial y}\ \text{at}\ y=\pm\eta, \tag{8}$$

$$\frac{\partial\psi}{\partial y}\pm\beta_1\left[(1+B)\frac{\partial^2\psi}{\partial y^2}-\frac{A}{3}\left(\frac{\partial^2\psi}{\partial y^2}\right)^3\right]=0,\quad \theta\pm\beta_2\frac{\partial\theta}{\partial y}=0,\ \phi\pm\beta_3\frac{\partial\phi}{\partial y}=0\ \text{at}\ y=\pm\eta. \tag{9}$$

Eliminating pressure by cross differentiation from Eqs. (4) and (5) ,we obtain

$$(1+B)\frac{\partial^4\psi}{\partial y^4}-\frac{A}{3}\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\psi}{\partial y^2}\right)^3-\left(M^2\right)\frac{\partial^2\psi}{\partial y^2}=0, \tag{10}$$

where A and B are the material parameters for the Eyring-Powell fluid model, δ is the wave number, E_1 is the wall elastance parameter, E_2 the mass per unit area parameter and E_3 is the wall damping parameter, $\Pr$ is the Prandtl number, Ec is the Eckert number, Sc is the Schmidt number and Sr is the Soret number, Re is the Reynolds number, θ is the temperature distribution and ϕ is the concentration distribution , Rd is the Radiation parameter.

III. PERTURBATION SOLUTION

The non-dimensional governing Eqs. (10), (6) and (7) are coupled and highly non-linear and their exact solutions may not be possible. Therefore, in order to obtain approximate series solutions, we can expand expressions for ψ, θ and ϕ in small Eyring-Powell fluid parameter A as follows:

$$\begin{aligned}\psi&=\psi_o+A\psi_1+O\left(A^2\right),\\ \theta&=\theta_o+A\theta_1+O\left(A^2\right),\\ \phi&=\phi_o+A\phi_1+O\left(A^2\right).\end{aligned} \tag{11}$$

3.1. Zeroth order system

Substituting Eq. (11) into Eqs. (10), (6) and (7) and boundary conditions (8) and (9), we get the following system

$$\psi_o=L_2\sinh(Ny)+L_3y \tag{12}$$

$$\theta_o=B_1+B_2y+L_{10}\cosh(2Ny)+L_{11}y^2 \tag{13}$$

$$\phi_o=C_1\cosh(N_2y)+C_2\sinh(N_2y)+L_{25}\cosh(2Ny)+L_{26} \tag{14}$$

3.2. First order system

Now using the solution expressions at the zeroth order into first order system and then solving the resulting problem, we have finally obtained

$$\psi_1=L_8\sinh(Ny)+L_6\sinh(3Ny)+L_7y\cosh(Ny) \tag{15}$$

$$\theta_1=B_3+B_4y+L_{14}\sinh(2Ny)+L_{20}\cosh(2Ny)+L_{21}\cosh(4Ny)+L_{22}y\sinh(2Ny)+L_{23}y^2 \tag{16}$$

$$\phi_1=C_3\cosh(N_2y)+C_4\sinh(N_2y)+L_{28}\sinh(2Ny)+L_{29}\cosh(2Ny)+L_{30}\cosh(4Ny)+L_{31}y\sinh(2Ny)+L_{32} \tag{17}$$

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IV. GRAPHICAL ANALYSIS

This section aims to analyze the obtained results graphically. The variations in velocity, temperature, concentration, stream lines, heat transfer coefficient and mass transfer coefficient caused by different physical parameters on the quantities of interest are discussed through Figs.2 – 6. We have adopted the following default fixed constants for numerical computations.

$$E_1 = 0.4,\ E_2 = 0.1,\ E_3 = 0.01,\ \varepsilon = 0.15,\ x = 0.2,\ B = 2, M = 1.0,\ \beta_1 = 0.01, A = 0.1,$$
$$t = 0.1,\ Br = 2, \beta_2 = 0.02,\ \Pr = 1,\ Rd = 1,\ Sc = 1, Sr = 1, \beta_3 = 0.02,\ x = 0.2$$

4.1. Flow characteristics

This section provides the variation of different physical parameters of interest on the axial velocity u , which is obtained using the expression $u = \frac{\partial \psi_o}{\partial y} + A\frac{\partial \psi_1}{\partial y}$. Fig. 2 shows that axial velocity diminishes with an increase in Hartmann number M because of the Lorentz force associated with the applied magnetic field along the transverse direction which opposes the flow and consequently the axial velocity decreases and the similar trend observed as the effect of Eyring-Powell fluid parameter B increases while opposite behavior observed from Figs. 3 – 4.

4.2. Heat characteristics

In this section, we have investigated the influence of various physical parameters of interest on the temperature distribution (θ) which is obtained using the expression $\theta = \theta_0 + A\theta_1$. From Fig. 5 – 7, It is found that in these figures the temperature is higher at the center of the channel as compared to the walls. This is in fact due to the consideration of viscous dissipation effects in the energy. The temperature decreases by increasing Eyring-Powell fluid parameter B , Hartmann number M , Radiation parameter Rd .

4.3. Mass characteristics

In this section, it is found the influence of various parameters of interest on the concentration distribution (ϕ) which is computed using the expression $\phi = \phi_0 + A\phi_1$ through Figs. 8 - 11. It is observed that in these figures apart from the temperature, concentration is smaller at the center of the channel as compared to the walls. The concentration diminishes with the effect of the magnitude of the higher values of A , M and opposite behavior observed in B

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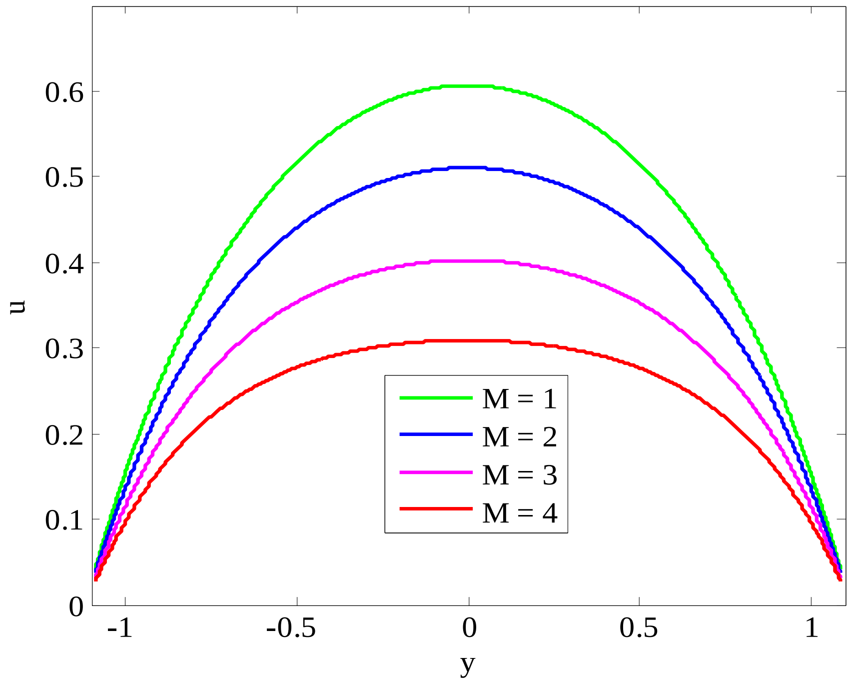

Fig.2. Effect of M on velocity profile.

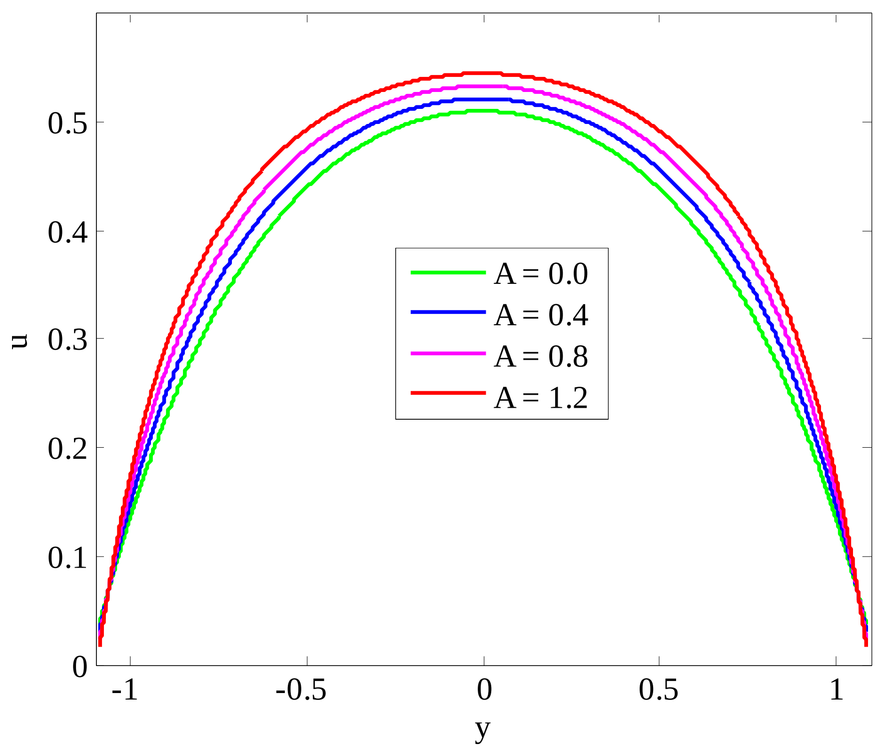

Fig.3. Effect of A on velocity profile.

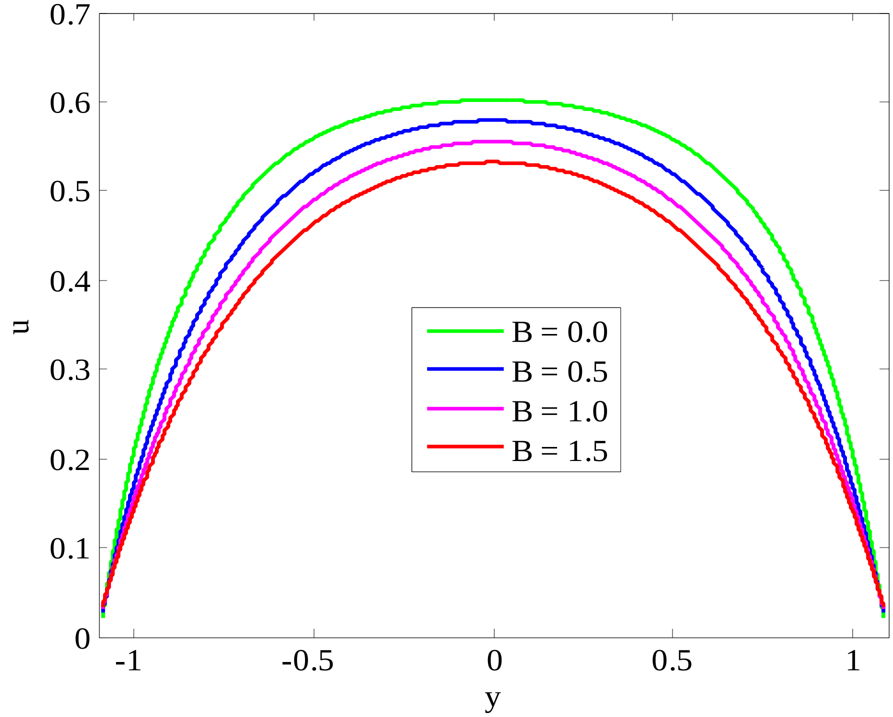

Fig.4. Effect of B *on velocity profile.*

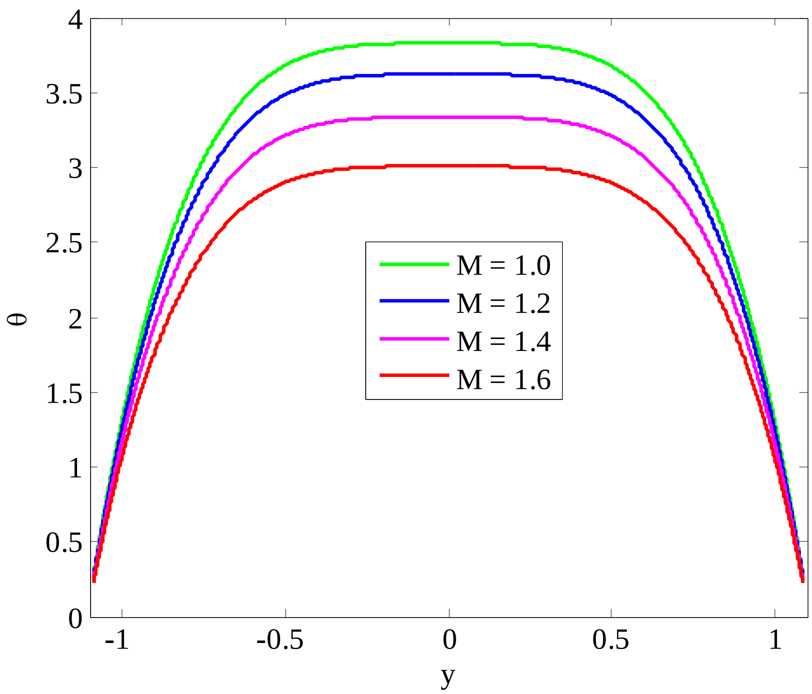

Fig.5. Effect of M *on temperature profile.*

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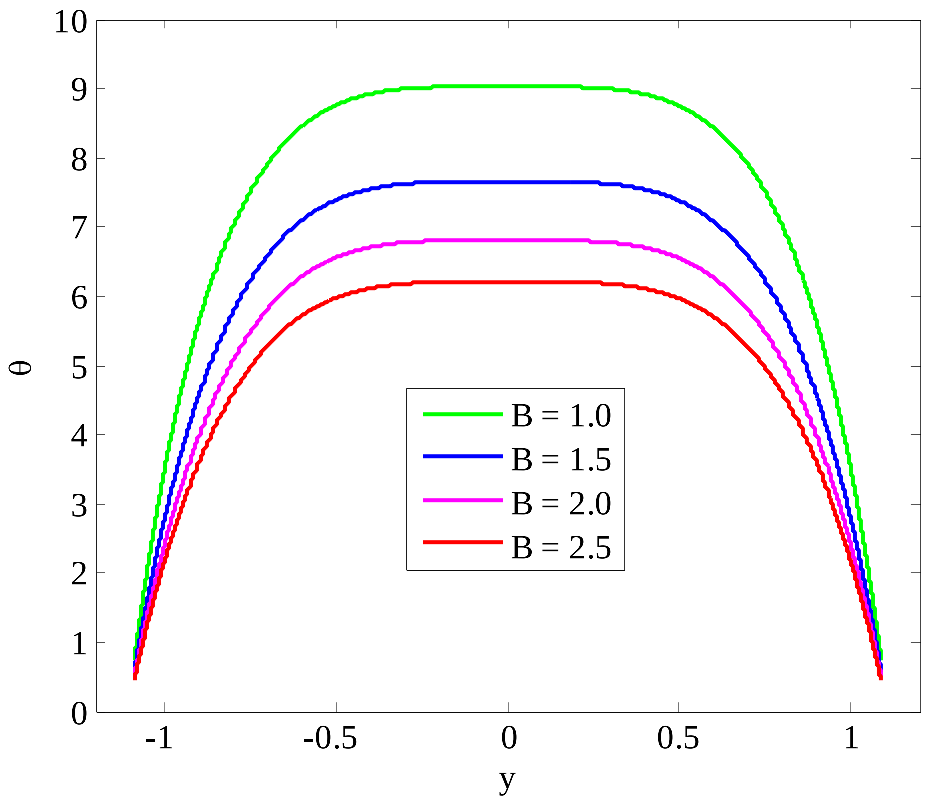

Fig.6. Effect of B on temperature profile.

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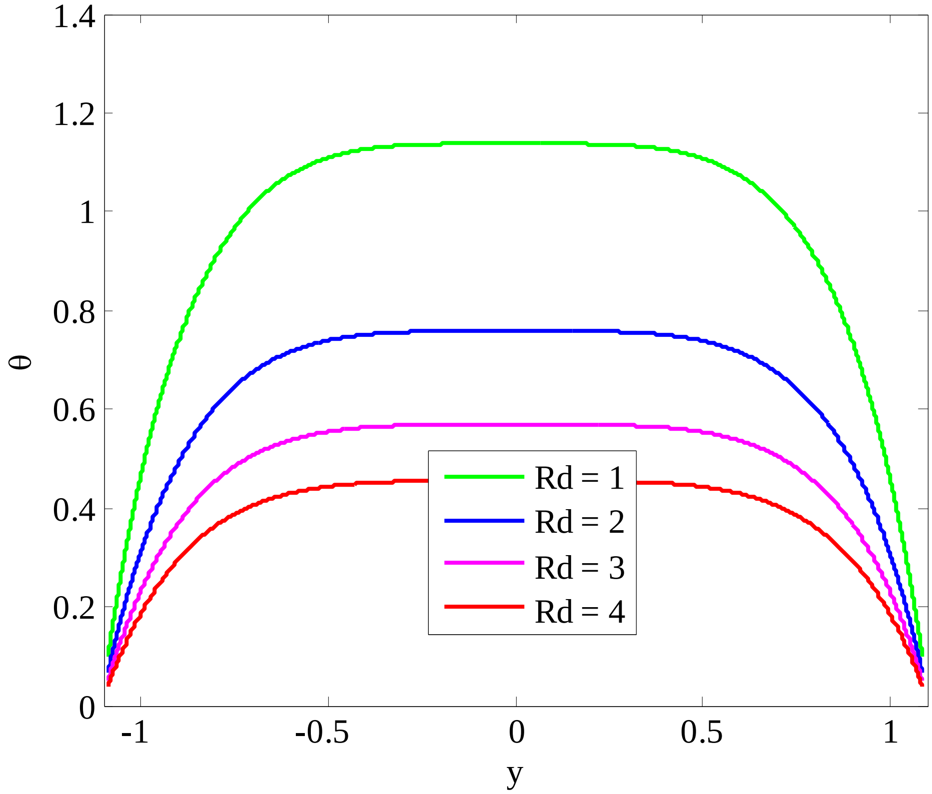

Fig.7. Effect of** Rd **on temperature profile.

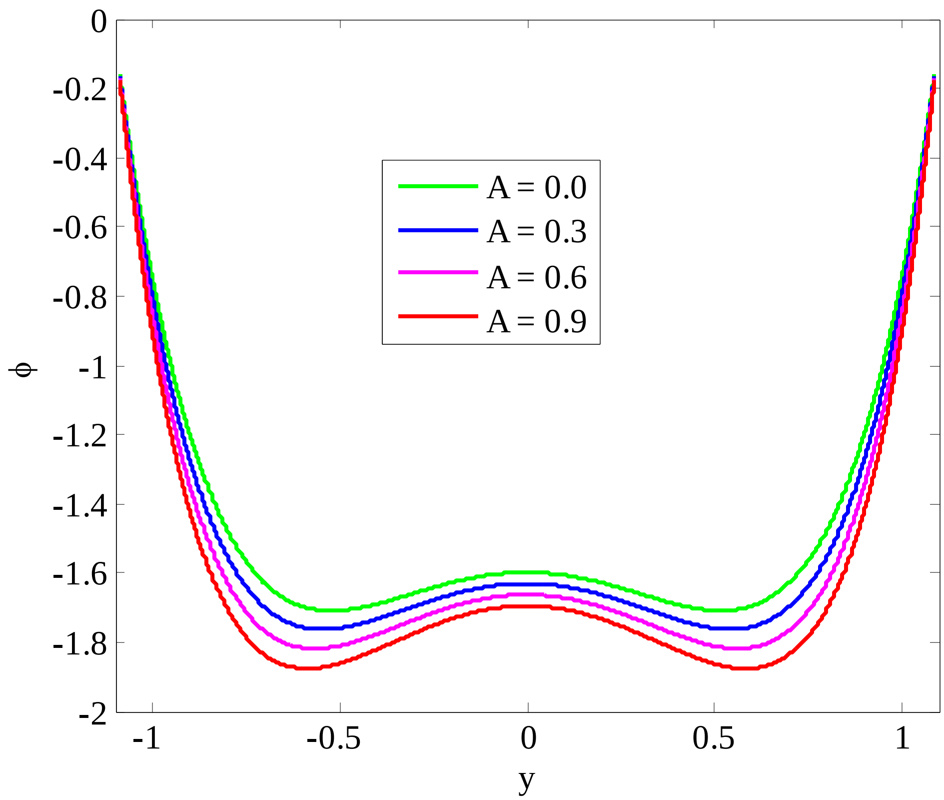

Fig.8. Effect of A ***on concentration profile.***

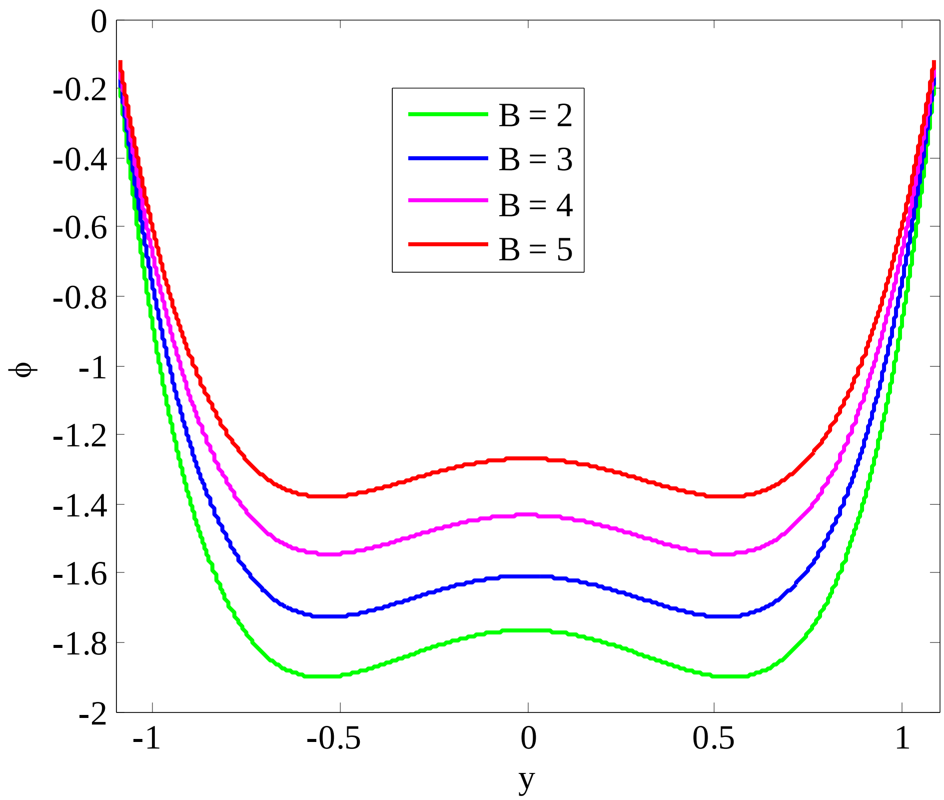

Fig.9. Effect of B on concentration profile

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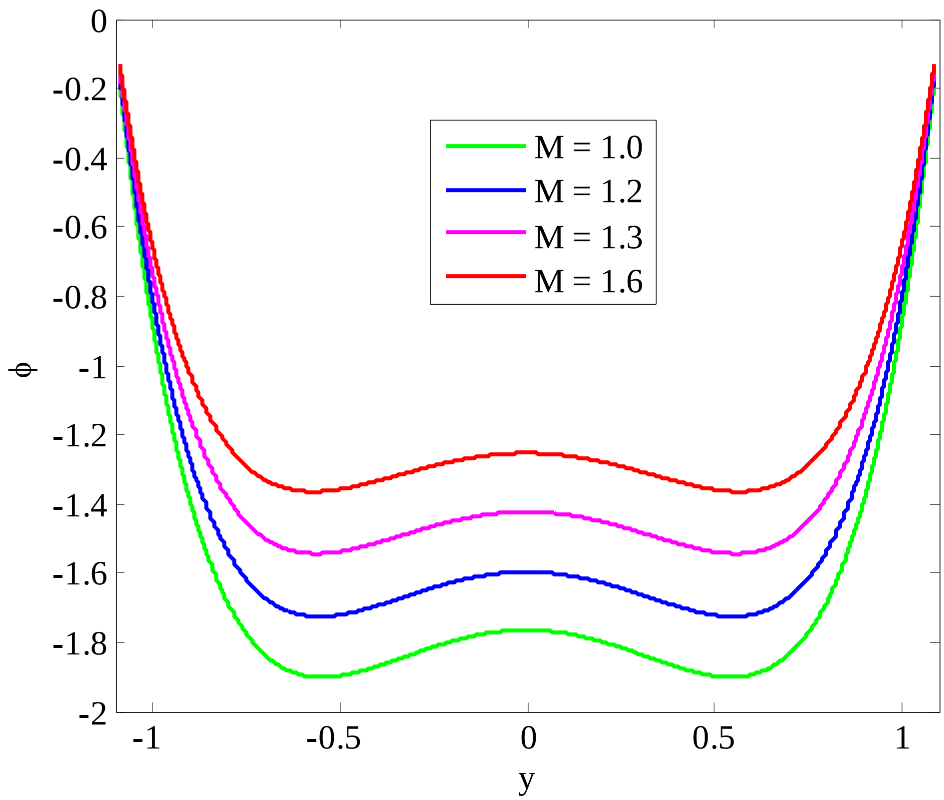

Fig.10. Effect of M ***on concentration profile.***

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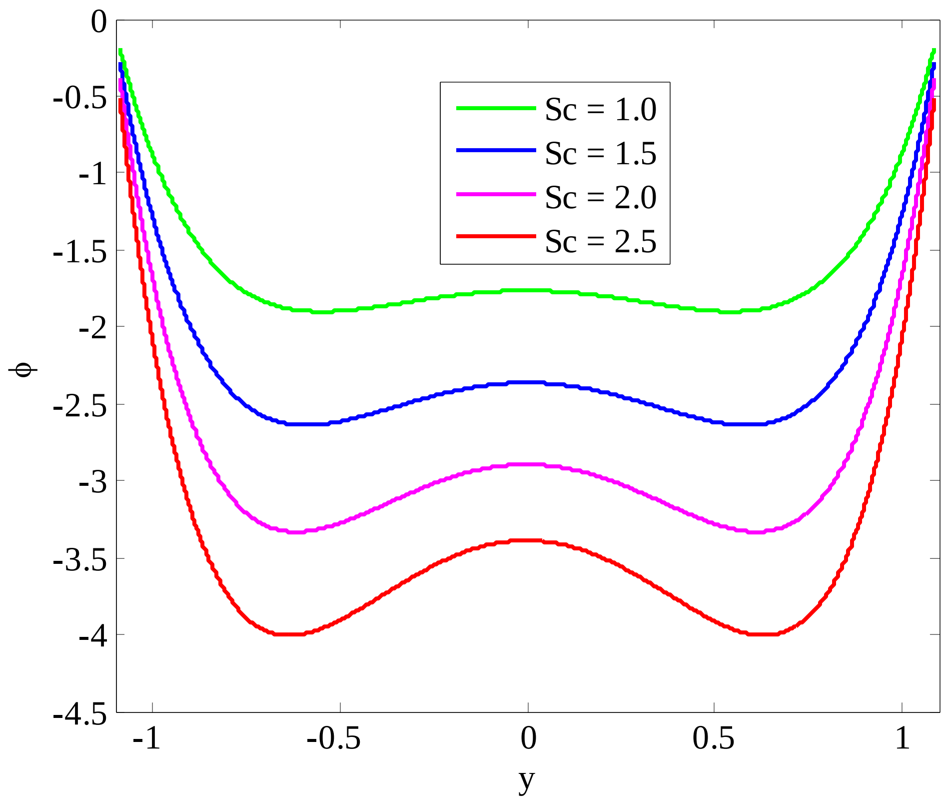

Fig.11. Effect of Sc on concentration profile

V. CONCLUDING REMARKS

In the present paper, we have investigated the MHD flow of radiation effects on peristaltic transport of Eyring-Powell fluid through a porous medium in a compliant wall channel with slip conditions. The governing equations are modeled using long wave length and low Reynolds number assumptions. The approximate solution is obtained using perturbation method for stream function, velocity, temperature, concentration. Our results can be summarized as follows:

1. The temperature depresses with an increasing in the radiation parameter Rd .
2. The concentration increases as larger values of chemical reaction parameter γ and radiation parameter Rd
3. If $Rd = 0$, our results are in agreement with Reference [8].

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