The Dynamics of an Eco-Epidemiological Model of three species with Holling type-II and Type-IV functional responses

To cite this article: S. Hariprasad *et al* 2021 *J. Phys.: Conf. Ser.* **1850** 012003

The Dynamics of an Eco-Epidemiological Model of three species with Holling type-II and Type-IV functional responses

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Abstract: In this paper, a prey-predator eco-epidemiological model with susceptible and diseased prey is constructed and investigated. The existence, uniqueness and boundedness of the solution of the system are studied. The conditions for the local and global stability of coexistence steady state are examined. At last, the mathematical results are interpreted with the help of numerical simulations.
Key words: - Infected, Susceptible, Boundedness, Local stability, Global stability, Routh-Hurwitze criteria.

1. Introduction:

Ecological systems like Prey- Predator, natural system, etc are dynamic, complex and non-linear in nature. The study of the dynamics of this relationship is one of the dominant subjects in mathematical Ecology, which can be obtained through the formulation and analysis of Mathematical models. Many researchers like Hastigs and Powell [2,9,14,16,17,21] examined the complex non-linear behavior of 3 species ecological models.

Mathematical epidemiology has become an interesting topic of research since the model of Kermack-McKendrick on SIRS (susceptible-infected-recovered-susceptible) systems. The effect of disease in ecological system is an important issue from mathematical and ecological point of view. Mathematical ecology and mathematical epidemiology are two different fields in the study of biology and applied mathematics. The combination of these two is studied which is termed as eco-epidemiology.

In this paper, we study the complex dynamics of a three-dimensional ecological model consisting of the species namely susceptible prey, infected prey and predator with functional responses. The functional response refers to the way a predator responds to the change in the density of prey attacked per unit time by the predator. In this model Type-II & Type-IV functional responses are considered to study the behavior of the systems.

Here the converted infected prey to susceptible will not be infected again which is the assumption we consider in this model. The predator will have interaction either with infected prey or susceptible prey, not both at a time.

The proposed model is

$$\begin{aligned}
\frac{dx_S}{dt} &= a_1 x_S - a_2 {x_S}^2 - \frac{m_0 x_S x_I}{\beta + x_S} + \eta_0 x_I - \frac{m_1 x_S x_P}{\gamma + {x_S}^2} \\
\frac{dx_I}{dt} &= \frac{m_0 x_S x_I}{\beta + x_S} - \eta_0 x_I + \frac{m_2 x_I x_P}{\alpha + x_I} \\
\frac{dx_P}{dt} &= \frac{m_3 x_S x_P}{\gamma + {x_S}^2} - \frac{m_2 x_I x_P}{\alpha + x_I} - \delta_0 x_P
\end{aligned} \tag{1.1}$$

Where $x_S(t), x_I(t), x_P(t)$ are denote the density of susceptible prey, infected prey and Predator respectively at any instant of time t. $a_1, a_2, \alpha, \beta, \gamma, m_0, m_2, m_1, m_3, \delta_o, \eta_0$ are positive constants. The parameter 'a_1' is growth rate of susceptible prey, The parameter 'a_2' is intra specific competition among individuals of prey x_S, The parameters α, β, γ are half saturation constants , m_0, m_2 are transmission rate from infected prey to susceptible prey, m_1, m_3 are the maximal growth rate of the species , η_0 is probability coefficient of converting infected prey into susceptible prey, δ_0 is mortality rate of the predator .

The biological meaning of system (1.1) is describes as follows

- In this food chain model, the population Susceptible (x_S) grows logistically in the absence of it's natural Predator (x_P) while it could be decreases due to hunting by the Predator (x_P) with the type-IV functional response. The susceptible Prey (x_S) is decreases due to interaction with Infected Prey (x_I).
- The infected prey (x_I) population grows when it interaction with either The susceptible Prey (x_S) or Predator (x_P) and it is having recovering rate (η_0).
- The Predator population (x_P) is decline in the absence of it's sole food source (x_S or x_I or both) and grows by hunting and eating susceptible Prey (x_S), The Predator population (x_P) is decreases due to interaction with Infected Prey (x_I).

The system (1.1) has eleven parameters. It is evident that dealing a system having more number of parameters is challenging and required more complicated analysis. Reformulating a model in dimensionless type is helpful from many aspects. This procedure will facilitate to see the consistency of the model equations and ensure that each one terms have an equivalent set of units in equation. Additionally, non-dimensionalizing a model reduces the amount of free parameters and divulges a smaller set of quantities that govern the dynamics.

After non-dimensionalization, The proposed model (1.1) becomes

$$\begin{aligned}
\frac{dx_S}{dt} &= \beta_1 x_S (1 - \mathrm{K}_1 x_S) - \frac{x_S x_I}{1 + x_S} - \beta_3 \frac{x_S x_P}{l_0 + {x_S}^2} + \beta_4 x_I \\
\frac{dx_I}{dt} &= \beta_5 x_I \left(\frac{x_S}{1 + x_S} - \mathrm{K}_2 \right) + \frac{x_I x_P}{1 + x_I} \\
\frac{dx_P}{dt} &= \beta_7 x_P \left(\frac{x_S}{l_0 + {x_S}^2} - \mathrm{K}_3 \right) - \beta_8 \frac{x_I x_P}{1 + x_I}
\end{aligned} \tag{1.2}$$

where $\beta_1 = \frac{a_1 \beta}{m_0 \alpha}; \beta_2 = \frac{a_2 \beta^2}{m_0 \alpha}; \beta_3 = \frac{m_1 \alpha}{m_2 \beta^2}; \beta_4 = \frac{\eta_0}{m_0}; \beta_5 = \frac{\beta}{\alpha}; \beta_6 = \frac{\eta_0 \beta}{m_0 \alpha}; \beta_7 = \frac{m_3}{m_0 \alpha};$

$$\beta_8 = \frac{m_2 \beta}{m_0 \alpha}; \beta_9 = \frac{\delta_o \beta}{m_0 \alpha}; \mathrm{K}_1 = \frac{a_2 \beta}{a_1}; \mathrm{K}_2 = \frac{\eta_0}{m_0}; \mathrm{K}_3 = \frac{\delta_0 \beta}{m_3}; l_0 = \frac{\gamma}{\beta^2}.$$

2. Boundedness of the system: In this section, we will attain some adequate conditions for the boundedness of the system.

Theorem(2.1): The system (1.2) is uniformly bounded.

Proof: define a function $\Psi(t) = x_S + x_I + x_P$, then

$$\frac{d\Psi}{dt} = \left(\beta_1 x_S - \mathrm{K}_1\beta_1 x_S{}^2\right) - \left(1-\beta_5\right)\frac{x_S x_I}{1+x_S} - \left(\beta_3 - \beta_7\right)\frac{x_S x_P}{l_0 + x_S{}^2} - \left(\beta_8 - 1\right)\frac{x_I x_P}{1+x_I} + \beta_4 x_I - \beta_5 x_I \mathrm{K}_2 - \beta_7 x_P \mathrm{K}_2$$

Choose $\beta_5 < 1,\ \beta_8 > 1,\ and\ \beta_3 > \beta_7$, then $\frac{d\Psi}{dt} \le \left(\beta_1 x_S - \mathrm{K}_1\beta_1 x_S{}^2\right) + \beta_4 x_I - \beta_5 x_I \mathrm{K}_2 - \beta_7 x_P \mathrm{K}_2$ .choose positive real number ξ such that $\frac{d\Psi}{dt} + \xi\Phi \le x_S\left(\beta_1 - \beta_1\mathrm{K}_1 x_S + \xi\right) \le \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1}$; where

$0 < \xi \le \min\left(\beta_5\mathrm{K}_2 - \beta_4, \beta_7\mathrm{K}_2\right)$ and $\frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1}$ is the maximum value of $x_S\left(\beta_1 - \beta_1\mathrm{K}_1 x_S + \xi\right)$.

Therefore $\frac{d\Psi(t)}{dt} + \xi\Psi(t) \le \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1}$.

From that we obtain $\Psi(t) \le \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1\xi} + \left(\Psi(0) - \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1\xi}\right)e^{-\xi t}\ for\, t \ge 0.$ As $t \to \infty$, then $\Psi(t) \le \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1\xi}$. Thus $0 \le \Psi(t) \le \frac{(\beta_1+\xi)^2}{4\beta_1\mathrm{K}_1\xi}$., Therefore, $\Psi(t)$ is bounded.

3. Steady states

The system has the following five steady state solutions resulting from $\frac{dx_S}{dt} = 0, \frac{dx_I}{dt} = 0, \frac{dx_P}{dt} = 0$.

$$1)\left(0,0,0\right) \qquad 2)\left(1/\mathrm{K}_2, 0, 0\right)$$

$$3)\, P\left(\bar{x}_S, \bar{x}_I, 0\right), where\, \bar{x}_S = \frac{\mathrm{K}_2}{1-\mathrm{K}_2}, \bar{x}_I = \frac{\left(1-\mathrm{K}_2\right)^2}{\beta_1\mathrm{K}_2\left(\beta_4 - \mathrm{K}_2\right)\left(\mathrm{K}_1\mathrm{K}_2 + \mathrm{K}_2 - 1\right)}$$

$$4)\ P\left(\hat{x}_S, 0, \hat{x}_P\right) where\, \hat{x}_S = \frac{1 + \left(1 - 4\mathrm{K}_3{}^2 l_0\right)^{\frac{1}{2}}}{2\mathrm{K}_3}, \hat{x}_P = \frac{\alpha_1\left(1-\mathrm{K}_1 x_S\right)\left(l_0 + x_S{}^2\right)}{\beta_3}$$

5) The coexistent steady state is obtained by from the equations:

$$\left[\beta_1\left(1-\mathrm{K}_1 x_S\right) - \beta_3\frac{x_P}{l_0 + x_S{}^2}\right] + x_I\left[\beta_4 - \frac{x_S}{1+x_S}\right] = 0 \qquad (3.1)$$

$$\left[\beta_5\left(\frac{x_S}{1+x_S} - \mathrm{K}_2\right) + \frac{x_P}{1+x_I}\right] = 0 \qquad (3.2)$$

$$\left[\beta_7\left(\frac{x_S}{l_0 + x_S{}^2} - \mathrm{K}_3\right) - \beta_8\frac{x_I}{1+x_I}\right] = 0 \qquad (3.3)$$

Solving equations (3.2) & (3.3),we get

$$\overset{*}{x}_I = \frac{\beta_7\left(x_S - \mathrm{K}_3(l_0 + x_S{}^2)\right)}{\left[\left(\beta_8 + \beta_7\mathrm{K}_3\right)\left(l_0 + x_S{}^2\right) - \beta_7 x_S\right]} \quad \text{and} \quad \overset{*}{x}_P = \frac{\beta_5\beta_8\left[\mathrm{K}_2 + x_S\left(\mathrm{K}_2 - 1\right)\right]\left[l_0 + x_S{}^2\right]}{\left(1 + x_S\right)\left[\left(\beta_8 + \beta_7\mathrm{K}_3\right)\left(m_0 + x_S{}^2\right) - \beta_7 x_S\right]}$$

By using $\overset{*}{x}_I$ and $\overset{*}{x}_P$, the equation (3.1) becomes

$$\begin{aligned}
&\beta_1\mathrm{K}_1\left(\beta_8 + \beta_7\mathrm{K}_3\right)x_S{}^5 + \beta_1\left[\left(\mathrm{K}_1 - 1\right)\left(\beta_8 + \beta_7\beta_3\right) - \beta_7\mathrm{K}_1\right]x_S{}^4 \\
&\quad + \left[\beta_1\left(\beta_8 + \beta_7\mathrm{K}_3\right)\left(l_0\mathrm{K}_1 - 1\right) + \beta_1\beta_7\left(1 - \mathrm{K}_1\right) + \beta_7\mathrm{K}_3\left(\beta_4 - 1\right)\right]x_S{}^3 \\
&\quad + \left[\beta_4\beta_7\left(\mathrm{K}_3 - 1\right) + \beta_3\beta_5\beta_8\left(\mathrm{K}_2 - 1\right) + \beta_7\left(\beta_1 - 1\right) + \beta_1 l_0\left(\mathrm{K}_1 - 1\right)\left(\beta_8 + \alpha_7\mathrm{K}_3\right)\right]x_S{}^2 \qquad (3.4) \\
&\quad + \left[\beta_8\left(\beta_3\beta_5\mathrm{K}_2 - \beta_1 m_0\right) + \beta_7 l_0\mathrm{K}_3\left(\beta_4 - \beta_1 - 1\right) - \beta_7\beta_4\right]x + \beta_4\beta_7\mathrm{K}_3 l_0 = 0.
\end{aligned}$$

By choosing the parameter values in either of the following two cases by Descartes' rule of sign, the equation (3.4) get a positive solution $\overset{*}{x}_S$. The two cases are

Case-1: If $\mathrm{K}_1 > 1, \mathrm{K}_2 > 1, \mathrm{K}_3 > 1, \beta_1 > 1, \beta_4 < 1, l_0 < 1, \beta_4 < (\beta_1 + 1), (\beta_3 + \frac{\beta_8}{\beta_7}) > \frac{\mathrm{K}_1}{\mathrm{K}_1 - 1}, \beta_3\beta_5\mathrm{K}_2 < \beta_1 l_0$,

Case-2: If $\mathrm{K}_1 < 1, \mathrm{K}_2 < 1, \mathrm{K}_3 > 1, \beta_1 < 1, \beta_4 > 1, l_0\mathrm{K}_1 > 1, \beta_4 < (\beta_1 + 1), \beta_3\beta_5\mathrm{K}_2 < \beta_1 l_0$.

Therefore, the positive equilibrium point is $E\left(\overset{*}{x}_S, \overset{*}{x}_I, \overset{*}{x}_P\right)$,where

$$\overset{*}{x}_I = \frac{\beta_7\left(\overset{*}{x}_S - \mathrm{K}_3(l_0 + \overset{*}{x}_S{}^2)\right)}{\left[\left(\beta_8 + \beta_7\mathrm{K}_3\right)\left(l_0 + \overset{*}{x}_S{}^2\right) - \beta_7\overset{*}{x}_S\right]}; \quad \overset{*}{x}_P = \frac{\beta_5\beta_8\left[\mathrm{K}_2 + \overset{*}{x}_S\left(\mathrm{K}_2 - 1\right)\right]\left[l_0 + \overset{*}{x}_S{}^2\right]}{\left(1 + \overset{*}{x}_S\right)\left[\left(\beta_8 + \beta_7\mathrm{K}_3\right)\left(m_0 + \overset{*}{x}_S{}^2\right) - \beta_7\overset{*}{x}_S\right]}$$

4. Stability analysis of coexistent steady state

Theorem(4.1): The interior steady state $E\left(\overset{*}{x}_S, \overset{*}{x}_I, \overset{*}{x}_P\right)$ is locally asymptotically stable, if $C_1 > 0,\ C_3 > 0$ and $\left(C_1C_2 - C_3\right) > 0$.

Proof:- The Jacobian matrix is $J\left(\overset{*}{x}_S, \overset{*}{x}_I, \overset{*}{x}_P\right) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & 0 \end{bmatrix}$ where

$$c_{11} = \beta_1\left(1 - 2\mathrm{K}_1\overset{*}{x}_S\right) - \frac{\overset{*}{x}_I}{(1 + \overset{*}{x}_I)^2} - \beta_3\overset{*}{x}_P\left(l_0 - \overset{*}{x}_S{}^2\right)\left(l_0 + \overset{*}{x}_S{}^2\right)^{-2}; c_{12} = \beta_4 - \frac{\overset{*}{x}_S}{1 + \overset{*}{x}_S}, c_{13} = -\beta_3\frac{\overset{*}{x}_S}{l_0 + \overset{*}{x}_S{}^2};$$

$$c_{21} = \frac{\beta_5\overset{*}{x}_I}{\left(1 + \overset{*}{x}_S\right)^2}; c_{22} = \frac{-\overset{*}{x}_I\overset{*}{x}_P}{\left(1 + \overset{*}{x}_I\right)^2}; c_{23} = \frac{\overset{*}{x}_I}{1 + \overset{*}{x}_I}; c_{31} = \beta_7\overset{*}{x}_P\left(m_0 - \overset{*}{x}_S{}^2\right)\left(m_0 + \overset{*}{x}_S{}^2\right)^{-2}; c_{32} = \frac{-\beta_8\overset{*}{x}_P}{\left(1 + \overset{*}{x}_S\right)^2}.$$

The characteristic equation of $J\left(E\right)$ is $\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$ .where

$C_1 = -\left(c_{11} + c_{22}\right), C_2 = \left(c_{11}c_{22} - c_{32}c_{23} - c_{12}c_{21} - c_{13}c_{31}\right); C_3 = c_{11}c_{32}c_{23} + c_{13}c_{31}c_{22} - c_{12}c_{23}c_{31} - c_{13}c_{21}c_{32}$.

Here $B_1 = \frac{\overset{*}{x}_I}{M_2^2}\left(1+\overset{*}{x}_P\right) + \beta_3 \overset{*}{x}_P (m_0 - \overset{*}{x}_S{}^2)\left(m_0 + \overset{*}{x}_S{}^2\right)^{-2} + \beta_1(2\mathrm{K}_1 \overset{*}{x}_S - 1)$,

$$B_3 = \frac{\beta_8 \overset{*}{x}_I \overset{*}{x}_P}{M_2^5 M_3^2}\left[\overset{*}{x}_I M_3^2 + \beta_3 \overset{*}{x}_P (l_0 - \overset{*}{x}_S{}^2) M_2^2 + \beta_1 (2\mathrm{K}_1 \overset{*}{x}_S - 1) M_2^2 M_3^2\right] + \left[\frac{\beta_7 \overset{*}{x}_I \overset{*}{x}_P (l_0 - \overset{*}{x}_S{}^2)(\overset{*}{x}_S - \beta_4 M_1)}{M_1 M_2 M_3^2}\right] +$$

$$\frac{\beta_3 \overset{*}{x}_S \overset{*}{x}_I \overset{*}{x}_P}{M_1^2 M_2^2 M_3^2}\left[\frac{\beta_7 \overset{*}{x}_P (m_0 - \overset{*}{x}_S{}^2) M_1^2}{M_3} - \beta_5 \beta_8 M_3\right].$$ Now consider $\Delta = C_1 C_2 - C_3$

$$\Delta = \frac{\overset{*}{x}_I \overset{*}{x}_P}{M_2^6 M_3^4}\left[\overset{*}{H}{}^2 + \overset{*}{H} M_3^2 (\overset{*}{x}_I \overset{*}{x}_P - M_2 \beta_8)\right] + \frac{\beta_5 \overset{*}{x}_I (\overset{*}{x}_S - M_1 \beta_4)(\overset{*}{H} + M_3^2 \overset{*}{x}_I \overset{*}{x}_P)}{M_1^3 M_2^2 M_3^2} + \frac{\beta_3 \beta_7 \overset{*}{x}_P \overset{*}{x}_S (l_0 - \overset{*}{x}_S{}^2)\overset{*}{H}}{M_2^2 M_3^3} +$$

$$\frac{\beta_8 \overset{*}{x}_I \overset{*}{x}_P}{M_2^2}\left[\frac{(\overset{*}{H} + M_3^2 \overset{*}{x}_I \overset{*}{x}_P)}{M_2^3 M_3^2} + \frac{\beta_3 \beta_5 \overset{*}{x}_S}{M_1^2 M_3}\right] + \frac{\beta_7 \overset{*}{x}_S \overset{*}{x}_I \overset{*}{x}_P (l_0 - \overset{*}{x}_S{}^2)}{M_2^2 M_3^2 M_1}\left[\beta_3 \overset{*}{x}_P M_1 M_3 - \frac{\beta_3 \overset{*}{x}_P M_1}{M_3} - \frac{(\overset{*}{x}_S - M_1 \beta_4) M_2}{\overset{*}{x}_S}\right]$$

where

$$M_1 = 1 + \overset{*}{x}_S; M_2 = 1 + \overset{*}{x}_I; M_3 = l_0 + \overset{*}{x}_S{}^2; \overset{*}{H} = \overset{*}{x}_I M_3^2 + \beta_3 \overset{*}{x}_P (l_0 - \overset{*}{x}_S{}^2) M_2^2 + \beta_1 (2\mathrm{K}_1 \overset{*}{x}_S - 1) M_2^2 M_3^2$$

$$\Delta = C_1 C_2 - C_3 > 0, \text{ if } i)\ 4\mathrm{K}_1^2 l_0 > 1\ ii)\ M_3^2 > 1\ iii)\ \beta_8 < \frac{M_1 \overset{*}{x}_P}{M_3}\left(\frac{\beta_7 \overset{*}{x}_I (l_0 - \overset{*}{x}_S{}^2)}{\beta_5 M_2}\right)^{\frac{1}{2}}.$$

By Routh-Hurwitze criteria, the steady state point $E\left(\overset{*}{x}_S, \overset{*}{x}_I, \overset{*}{x}_P\right)$ is locally asymptotically stable, if $C_1 > 0, C_3 > 0$ *and* $\left(C_1 C_2 - C_3\right) > 0$ holds.

Theorem(4.2): Along with the conditions stated in the above theorems(4.1) and if $\beta_3 < \frac{(l_0 + \overset{*}{x}_S{}^2)(l_0 + x_S{}^2)}{\overset{*}{x}_P\left(x_S + \overset{*}{x}_S\right)}\left[\frac{\beta_4 \overset{*}{x}_I}{x_S \overset{*}{x}_S} - \frac{\overset{*}{x}_I}{(1 + x_S)(1 + \overset{*}{x}_S)} + \beta_1 k_1\right]$ then , the steady state $E\left(\overset{*}{x}_S, \overset{*}{x}_I, \overset{*}{x}_P\right)$ is globally asymptotically stable.

Proof: Consider a Lyapunov function $v(t)$ such that

$v(t) = n_1\left[x_S - \overset{*}{x}_S - \overset{*}{x}_S \ln\left(\frac{x_S}{\overset{*}{x}_S}\right)\right] + n_2\left[x_I - \overset{*}{x}_I - \overset{*}{x}_I \ln\left(\frac{x_I}{\overset{*}{x}_I}\right)\right] + n_3\left[x_P - \overset{*}{x}_P - \overset{*}{x}_P \ln\left(\frac{x_P}{\overset{*}{x}_P}\right)\right]$, where n_1, n_2 and n_3 are positive constants.

$$\frac{dv}{dt} = n_1\left[\frac{\overset{*}{x}_I}{(1 + x_S)(1 + \overset{*}{x}_S)} + \frac{\beta_3 \overset{*}{x}_P (x_S + \overset{*}{x}_S)}{(l_0 + x_S{}^2)(l_0 + \overset{*}{x}_S{}^2)} - \frac{\beta_4 \overset{*}{x}_I}{x_S \overset{*}{x}_S} - \beta_1 \mathrm{K}_1\right]\left(x_S - \overset{*}{x}_S\right)^2 - \frac{n_2 \overset{*}{x}_P}{(1 + x_I)(1 + \overset{*}{x}_I)}\left(x_I - \overset{*}{x}_I\right)^2$$

$$+\frac{1}{(1+x_S)}\left[\frac{n_1}{x_S}(\beta_4(1+x_S)-x_S)+\frac{n_2\beta_5}{(1+x_S^*)}\right]\left(x_S-x_S^*\right)\left(x_I-x_I^*\right)+\frac{1}{(l_0+x_S^2)}\left[n_3\beta_7-n_1\beta_3-\frac{n_3\beta_7 x_S^*(x_S+x_S^*)}{(l_0+x_S^{*2})}\right].\times.$$

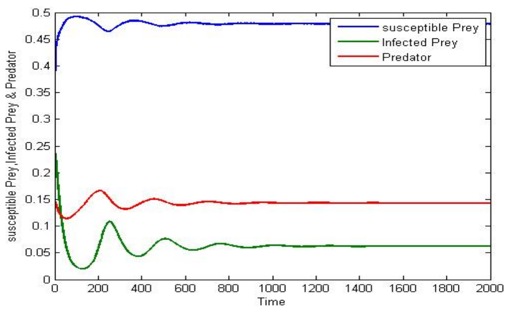
$$\left(x_S-x_S^*\right)\left(x_P-x_P^*\right)+\frac{1}{1+x_I}\left[n_2-n_3\beta_8+\frac{n_3\beta_8 x_I^*}{(1+x_I^*)}\right]\left(x_P-x_P^*\right)\left(x_I-x_I^*\right).$$

Chose non-negative constants $n_1=n_2=1, n_3=\frac{\left(1+x_I^*\right)}{\beta_8}, \beta_4=\frac{x_S\left(1+x_S^*-\beta_5\right)}{(1+x_S^*)(1+x_S)}, \beta_7=\frac{\beta_3\beta_8(l_0+x_S^{*2})}{\beta_7\left(1+x_I^*\right)}$

and $\beta_3<\frac{(l_0+x_S^{*2})(l_0+x_S^2)}{x_P^*\left(x_S+x_S^*\right)}\left[\frac{\beta_4 x_I^*}{x_S x_S^*}-\frac{x_I^*}{(1+x_S)(1+x_S^*)}+\beta_1 k_1\right]$,then $\frac{dv}{dt}<0$. Hence, The point $E\left(x_S^*, x_I^*, x_P^*\right)$ is globally asymptotically stable, if the above conditions are satisfied.

5. Numerical Simulations:

If choose the parameter values $l_0=0.009$; $\beta_1=1.09$; $\beta_3=0.09$; $\beta_4=0.09$; $\beta_5=0.19$; $\beta_7=0.009$; $\beta_8=0.09$; $K_1=1.9$; $K_2=1.00009$; $K_3=1.4$;Then we obtain the positive Steady state point is (0.5, 0.02, 0.0016) and $C_1=0.1630>0$; $C_2=0.0049$; $C_3=0.00000082966>0$; $C_1C_2-C_3=0.00079662>0$ .Therefore the system (1.2) is stable at the point (0.5, 0.02, 0.0016). The corresponding stability graphs as follows



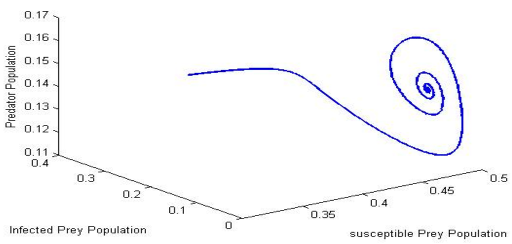


Figure.1

Figure. 2

Figure.1 represents the time Vs populations x_S, x_I, x_P and **Figure.2** represents the phase portraits of x_S, x_I, x_P.

If choose the parameter values $l_0=0.0998$; $\beta_1=1.07$; $\beta_3=0.009$; $\beta_4=0.08$; $\beta_5=0.2$; $\beta_7=0.01$; $\beta_8=0.099$; $K_1=1.42$; $K_2=1.01$; $K_3=1.2$;Then we obtain the positive Steady state point is

(0.7, 0.0184, 0.0003) and $C_1 = 0.1630$ >0 ; $C_2 = 0.0049$; $C_3 = 0.00000082966$>0 ; $C_1C_2 - C_3 =$ 0.00079662 >0 .Therefore the system (1.2) is stable at the point (0.7, 0.0184, 0.0003). The corresponding stability graphs as follows

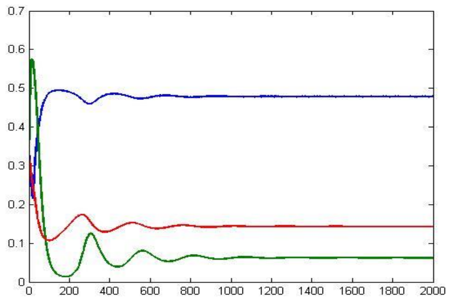


Figure. 3

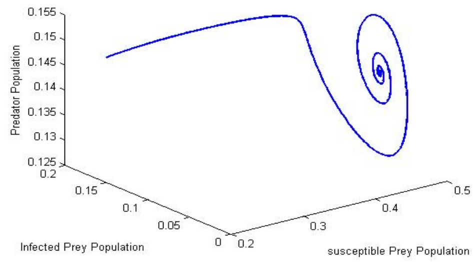


Figure. 4

Figure.3 represents the time Vs populations x_S, x_I, x_P and **Figure.4** represents the phase portraits of x_S, x_I, x_P.

6. Conclusions:

In this paper, a predator and two preys out of which one is susceptible and another one is considered as infected. The boundedness of the solutions and existence of steady states is established in the **sections 2 & 3**. The local and global stability of the proposed model around its steady states has been analyzed. The numerical simulations are carried by using MATLAB.

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