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A MATHEMATICAL ANALYSIS OF THE SIR MODEL WITH HOLLING TYPE II FUNCTIONAL OCCURRENCE RATE AND TREATMENT RATE

*N.Phani Kumar[1], K.Siva Reddy[2], M.A.Srinivas[3], K.V.L.N.Acharyulu[4]
[1]Department of Basic Sciences and Humanities,
Vignan Institute of Technology and Science, Hyderabad, India
[2]Department of Mathematics, Anurag University, Venkatapur, Hyderabad, India
[3]Department of Mathematics, Jawaharlal Nehru Technological University, JNTUH, Hyderabad
[4]Department of Mathematics, Bapatla Engineering College, Bapatla, India
(*Corresponding Author Email: **nphanikumar20@gmail.com**)

ABSTRACT

In this paper, a SIR epidemic model is proposed with nonlinear inhibitory effect and saturated treatment rate. When deciding the threshold value for the disease and the model's dynamics, the basic reproduction number is determined. The requirements for the existence of all equilibrium points are known and we have also found that they depend on the conditions. In local and global terms, the stability of equilibrium is discussed. The stochastic version of the model was also formulated to take into account the effect of noise. Population variance intensities (fluctuations) around the positive point of equilibrium due to noise have been measured. Every attempt was made to present the numerical simulations we proposed for the model.It is explicitly expected that the theoretical results will be supported and tested.

Key words: Local Stability, Global Stability, SIR epidemic model.

1. INTRODUCTION

Different types of widespread models have been mapped, analysed and applied to a multi various infectious diseases. With qualitative and quantitative criteria, all these have been carried out. Epidemic models, however, are important in the effective study of the widespread use and management of infectious diseases. Moreover, these models are critical in policy making, including optimising the assessment of multiple detection control programmes. In many disease models [3, 5, 8, 9], the bilinear incidence rate is also enforced. It is observed that the overcrowding of infected or susceptible individuals saturates the number of active interactions between infectious and susceptible individuals.

Saturated incidence rate [1]: $g\left(I\right)S=\dfrac{\beta IS}{S+\rho I}$ where an inhibitory effect that explains the saturation effects or psychological effects is calculated by the positive constant [3, 5, 6]. In addition, care for a disease plays a crucial role in the prevention or minimization of the transmission of several types of infectious diseases.The treatment of a disease in any group or nation is limited in general. A good disease model therefore represents the consideration of an efficient treatment rate. In the classical SIR model, a constant removal rate was taken into account by Wang and Ruan13].They carried out a study of stability and the delay in treatment.Many mathematicians opened new eras [1, 2, 4, 5, 6, 7, 8, 10, 13, and 14] with innovative ideas on these SIR epidemic models.

The mathematical model is constituted in section 2. Section 3 dealt with equilibrium and its life. In addition, the most significant amount, i.e. the simple reproduction number, is calculated in the modelling of the disease. The local and global stability of equilibrium points were evaluated in section 4.Section 5 is dedicated to the model's stochastic variant. The numerical simulations are carried out for supporting these results.

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2. THE MATHEMATICAL MODEL

The recovered individuals via treatment in the following considered SIR model is assumed that they gained permanent immunity. As stated in the introduction the incidence rate and the treatment rate was taken as Holling type –II. To construct an SIR model, we divide population into three subdivisions i.e. Susceptible (S), Infectious (I) and Recovered (R). As follows the model to be analysed is:

$$\begin{aligned}\frac{dS}{dt} &= b-\mu S-\frac{\beta IS}{S+\rho I}+\delta R\\ \frac{dI}{dt} &= \frac{\beta IS}{S+\rho I}-\mu I-\delta I-\frac{\alpha I}{1+\delta I} \qquad \text{........(1)}\\ \frac{dR}{dt} &= \gamma I-\mu R-\delta R+\frac{\alpha I}{1+\delta I}\end{aligned}$$

Here b is the rate of recruitment rate, ρ the inhibitory effect, β is the force of infection at which a susceptible individual is exposed and transmission duration is $\frac{1}{\beta}$.

Individuals become contagious after an incubation period and pass into infectious compartments. Here μ refers death rate and ρ refers inhibitory effect. γ Stands for the recovery of infective Here the parameters $b,\beta,\mu,\gamma,\alpha$ are all positive and ρ and δ are nonnegative.

3. EQUILIBRIUM POINTS

The equilibrium points are the solutions of the system $\frac{dS}{dt}=0$, $\frac{dI}{dt}=0$ and $\frac{dR}{dt}=0$. System (1) has the following equilibrium points.

- The infected and recovery free equilibrium point is $E_1=\left(S^{\bullet},0,0\right)$, where $S^{\bullet}=\frac{b}{\mu}$.
- The interior equilibrium point is $E^*=\left(S^*,I^*,R^*\right)$, where

  $$S^*=\frac{\left[(\mu+\gamma)(1+\delta I^*)+\alpha\right]\rho I^*}{\beta(1+\delta I^*)-(\mu+\gamma)(1+\delta I^*)-\alpha},\ R^*=\frac{I^*\left(\gamma+\gamma\delta I^*+\alpha\right)}{(\mu+\gamma)(1+\delta I^*)} \text{ exists if } \beta>(\mu+\gamma)+\frac{\alpha}{(1+\delta I^*)}$$

  and I^* can be determined from the equation $\frac{b}{S^*}-\mu-\frac{\beta I^*}{S^*+\rho I^*}+\frac{\delta R^*}{S^*}=0.$

4. STABILITY ANALYSIS

In this section, the stability of the system (1) is investigated around the equilibrium points. Now, the jacobian matrix of the system (1) is given by

$$J(S,I,R)=\begin{bmatrix} -\mu-\frac{\beta\rho I^2}{(S+\rho I)^2} & -\frac{\beta S^2}{(S+\rho I)^2} & \delta \\ -\frac{\beta\rho I^2}{(S+\rho I)^2} & \frac{\beta S^2}{(S+\rho I)^2}-(\mu+\gamma)-\frac{\alpha}{(1+\delta I)^2} & 0 \\ 0 & \gamma+\frac{\alpha}{(1+\delta I)^2} & -(\mu+\delta)\end{bmatrix}$$

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Theorem 1: The infected and recovery free equilibrium point E_1 is locally asymptotically stable if $R_0 < 1$.

Proof: The jacobian matrix of $J(S^{\bullet},0,0)$ is given by

$$J(S^{\bullet},0,0)=\begin{bmatrix} -\mu & -\beta & \delta \\ 0 & \beta-\mu-\gamma-\alpha & 0 \\ 0 & \gamma+\alpha & -(\mu+\delta) \end{bmatrix}$$

The characteristic equation of above matrix is $(\mu+\lambda)\left[(-\beta+\mu+\gamma+\alpha+\lambda)(\mu+\delta+\lambda)\right]=0$.

The roots of the above characteristic equation are $\lambda=-\mu,\ \beta-\mu-\gamma-\alpha,\ -\mu-\delta$. let $R_0=\dfrac{\beta}{\mu+\gamma+\alpha}<1$. Thus, the equilibrium point E_1 is locally asymptotically stable if $R_0 < 1$.

Theorem 2: The interior equilibrium point E^* is conditionally locally asymptotically stable.

Proof: The jacobian matrix of $J(S^*,I^*,R^*)$ is given by

$$J(S^*,I^*,R^*)=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}, \text{ where}$$

$$a_{11}=-\mu-\frac{\beta\rho I^2}{(S+\rho I)^2};\ a_{12}=-\frac{\beta S^2}{(S+\rho I)^2};\ a_{13}=\delta$$

$$a_{21}=-\frac{\beta\rho I^2}{(S+\rho I)^2};\ a_{22}=-\frac{\beta\rho SI}{(S+\rho I)^2}+\frac{\alpha\delta I}{(1+\delta I)^2};\ a_{23}=0$$

$$a_{31}=0;\ a_{32}=\gamma+\frac{\alpha}{(1+\delta I)^2};\ a_{33}=-(\mu+\delta).$$

The characteristic equation of $J(S^*,I^*,R^*)$ is $\lambda^3+(A_1)\lambda^2+(A_2)\lambda+(A_3)=0$. Where

$$A_1=-(a_{11}+a_{22}+a_{33})=\left[\mu+\delta+\frac{\beta\rho SI}{(S+\rho I)^2}-\frac{\alpha\delta I}{(1+\delta I)^2}+\mu+\frac{\beta\rho I^2}{(S+\rho I)^2}\right];$$

$$A_2=a_{22}a_{33}+a_{11}a_{22}+a_{11}a_{23}-a_{12}a_{21}=$$

$$\left[\left(\mu+\frac{\beta\rho I^2}{(S+\rho I)^2}\right)\left(\mu+\delta+\frac{\beta\rho SI}{(S+\rho I)^2}-\frac{\alpha\delta I}{(1+\delta I)^2}\right)+\left((\mu+\delta)\left(\frac{\beta\rho SI}{(S+\rho I)^2}-\frac{\alpha\delta I}{(1+\delta I)^2}+\frac{\beta^2\rho S^2 I^2}{(S+\rho I)^4}\right)\right)\right]$$

$$A_3=a_{12}a_{21}a_{33}-a_{11}a_{22}a_{33}-a_{13}a_{21}a_{32}=\left[\begin{array}{l}\left(\mu+\dfrac{\beta\rho I^2}{(S+\rho I)^2}\right)(\mu+\delta)\left(\dfrac{\beta\rho SI}{(S+\rho I)^2}-\dfrac{\alpha\delta I}{(1+\delta I)^2}\right)+\\ (\mu+\delta)\left(\dfrac{\beta\rho I^2}{(S+\rho I)^2}\right)\left(\dfrac{\beta S^2}{(S+\rho I)^2}\right)-\left(\gamma\delta+\dfrac{\alpha\delta}{(1+\delta I)^2}\right)\left(\dfrac{\beta\rho I^2}{(S+\rho I)^2}\right)\end{array}\right]$$

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By Rowth-Hurwitze criteria, the interior equilibrium point E^* is locally asymptotically stable, if $A_1 > 0,\ A_3 > 0$ and $(A_1A_2 - A_3) > 0$.

Theorem 3: The interior equilibrium point E^* is globally asymptotically stable.
Proof: Let we define the positive function

$v(t) = l_1\left(S - S^* - S^*\log(\frac{S}{S^*})\right) + l_2\left(I - I^* - I^*\log(\frac{I}{I^*})\right) + l_3\left(R - R^* - R^*\log(\frac{R}{R^*})\right)$, where l_1, l_2 *and* l_3 are positive constants.

$$\frac{dv}{dt} = l_1\left(\frac{S-S^*}{S}\right)\frac{dS}{dt} + l_2\left(\frac{I-I^*}{I}\right)\frac{dI}{dt} + l_3\left(\frac{R-R^*}{R}\right)\frac{dR}{dt}$$

$$\frac{dv}{dt} = \left(S-S^*\right)\left[\frac{b}{S} + \frac{\beta I}{S+\rho I} + \frac{\delta R}{S} - \frac{b}{S^*} - \frac{\beta I^*}{S^*+\rho I^*} - \frac{\delta R^*}{S^*}\right]$$
$$+ l_1\left(I-I^*\right)\left[\frac{\beta S}{S+\rho I} - \frac{\alpha}{1+\delta I} - \frac{\beta S^*}{S^*+\rho I^*} + \frac{\alpha}{1+\delta I^*}\right]$$
$$+ l_2\left(R-R^*\right)\left[\frac{\gamma I}{R} + \frac{\alpha I}{R\left(1+\delta I\right)} - \frac{\gamma I^*}{R^*} - \frac{\alpha I^*}{R^*\left(1+\delta I^*\right)}\right]$$

$$\frac{dv}{dt} = \left(S-S^*\right)\left[b\left(\frac{1}{S} - \frac{1}{S^*}\right) + \delta\left(\frac{R}{S} - \frac{R^*}{S^*}\right) + \beta\left(\frac{I}{S+\rho I} - \frac{I^*}{S^*+\rho I^*}\right)\right]$$

$$+ l_1\left(I-I^*\right)\left[\beta\left(\frac{S}{S+\rho I} - \frac{S^*}{S^*+\rho I^*}\right) - \alpha\left(\frac{1}{1+\delta I} - \frac{1}{1+\delta I^*}\right)\right]$$

$$+ l_2\left(R-R^*\right)\left[\gamma\left(\frac{I}{R} - \frac{I^*}{R^*}\right) + \alpha\left(\frac{I}{R\left(1+\delta I\right)} - \frac{I^*}{R^*\left(1+\delta I^*\right)}\right)\right]$$

$$\frac{dv}{dt} = -\left[\frac{b+\delta R^*}{SS^*} + \frac{\beta I^*}{\left(S+\rho I\right)\left(S^*+\rho I^*\right)} - \frac{\delta}{2S} - \frac{\beta S^* - l_1\beta\rho I^*}{2\left(S+\rho I\right)\left(S^*+\rho I^*\right)}\right]\left(S-S^*\right)^2$$

$$-\left[\begin{array}{l}\frac{l_1\beta\rho S^*}{\left(S+\rho I\right)\left(S^*+\rho I^*\right)} - \frac{\beta S^*}{2\left(S+\rho I\right)\left(S^*+\rho I^*\right)} - \frac{l_1\beta\rho I^*}{2\left(S+\rho I\right)\left(S^*+\rho I^*\right)} - \frac{l_1\alpha\delta}{\left(1+\delta I\right)\left(1+\delta I^*\right)} \\ \qquad - \frac{l_2\gamma}{2R} - \frac{l_2\alpha}{2R\left(1+\delta I\right)\left(1+\delta I^*\right)}\end{array}\right]\left(I-I^*\right)^2$$

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$$-\left[\frac{l_2\gamma I^*}{RR^*}+\frac{l_2(I^*\alpha+\alpha\delta II^*)}{RR^*(1+\delta I)(1+\delta I^*)}-\frac{\delta}{2S}-\frac{l_2\gamma}{2R}-\frac{l_2\alpha}{2R(1+\delta I)(1+\delta I^*)}\right](R-R^*)^2.$$

If $S^*<\dfrac{\delta}{2b};\ \rho<\dfrac{2}{l_1}-\dfrac{S^*}{l_1I^*};\ \gamma>\dfrac{R\delta R^*}{l_2S\left(2I^*-R^*\right)};\ I>\dfrac{1}{\delta}\left(\dfrac{R^*}{2I^*}-1\right)$, then $\dfrac{dv}{dt}<0$.

Hence, The interior equilibrium point E^* is globally asymptotically stable if, it satisfies the above conditions.

5. STABILITY GRAPHS

(1). b=50; μ = 0.00757; β = 0.12; ρ = 0.0999192; δ = 0.080115; γ = 0.09998; α = 0.002

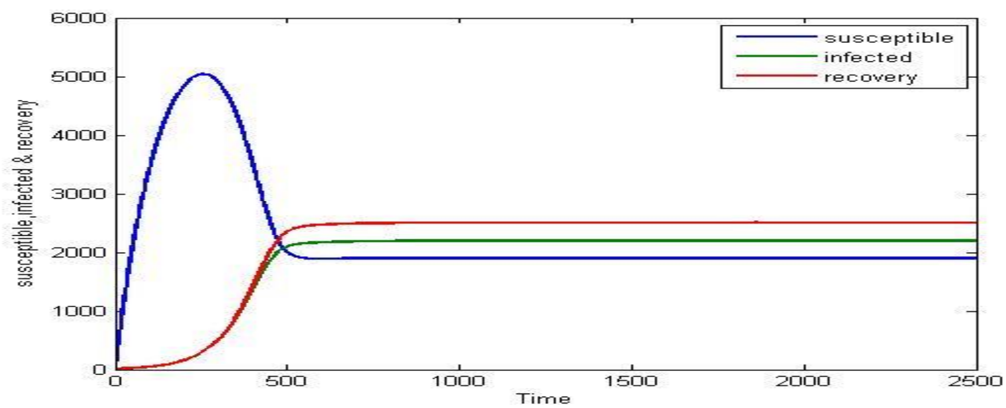

Fig.1

(2). b =50; μ = 0.00757; β = 0.12; ρ = 0.0999192; δ = 0.0995; γ = 0.09998; α = 0.002

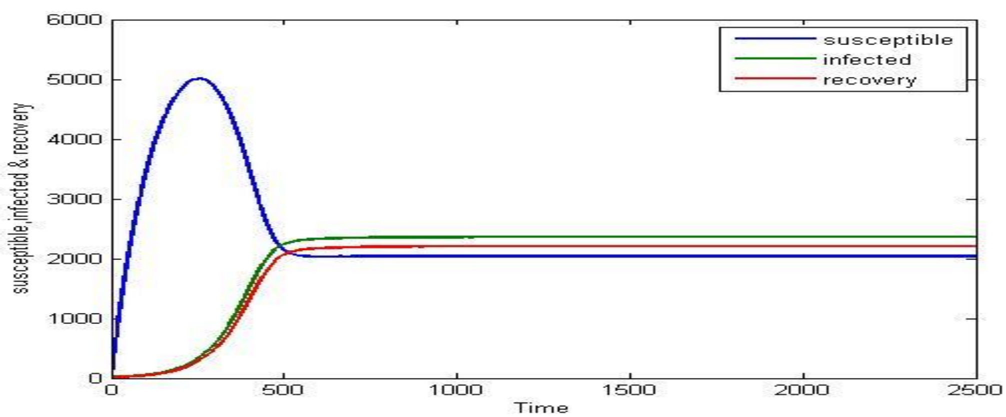

Fig.2

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(3). b =50; μ = 0.00757; $\beta = 0.12$; ρ = 0.0999192; δ = 0.0.05; $\gamma = 0.09998$; $\alpha = 0.002$

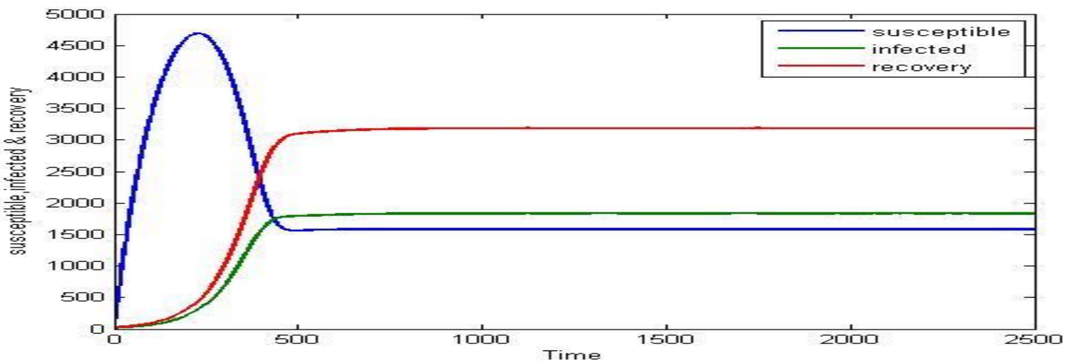

Fig.3

(4). b =50; μ = 0.009757; $\beta = 0.12$; ρ = 0.0999192; δ = 0.05; $\gamma = 0.09998$; $\alpha = 0.002$

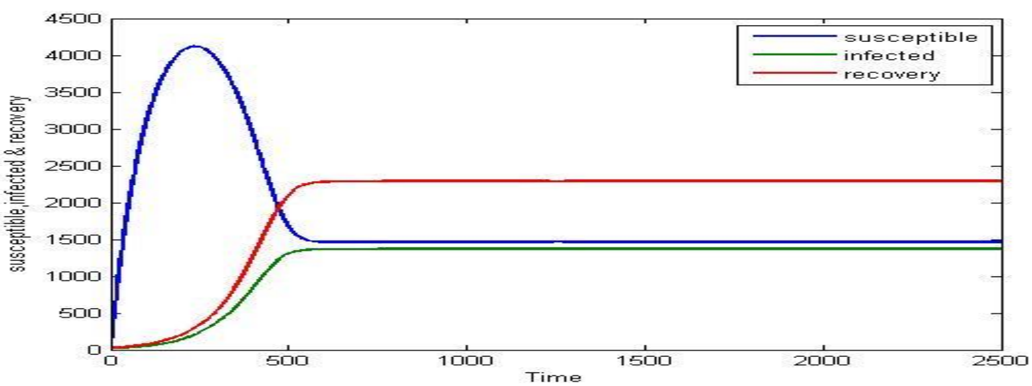

Fig.4

(5). b =50; μ = 0.009757; $\beta = 0.12$; ρ = 0.0999192; δ = 0.0995; $\gamma = 0.09998$; $\alpha = 0.0002$

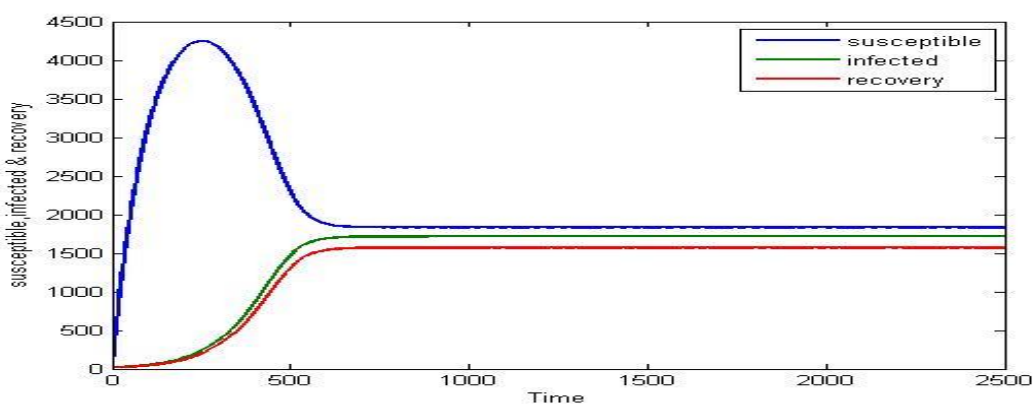

Fig.5

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(6).b =50; $\mu = 0.00559757; \beta = 0.12; \rho = 0.0999192; \delta = 0.0995; \gamma = 0.09998; \alpha = 0.0112$

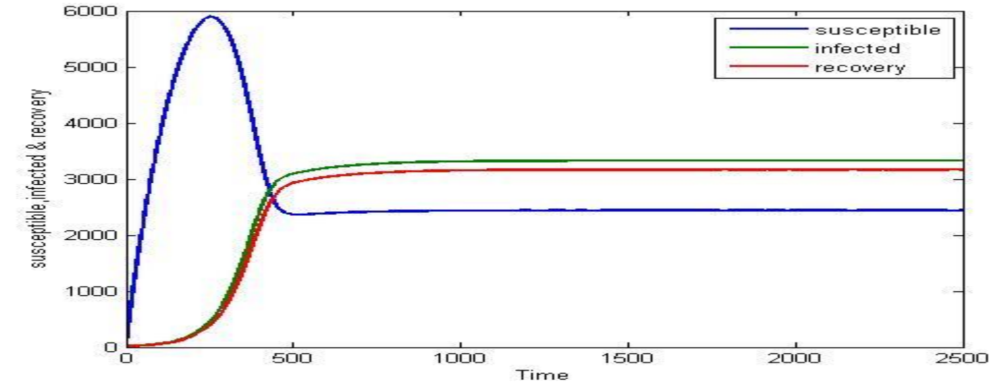

Fig.6

(7).b =50; $\mu = 0.00559757; \beta = 0.12; \rho = 0.0999192; \delta = 0.090095; \gamma = 0.09998;$
$\alpha = 0.012$

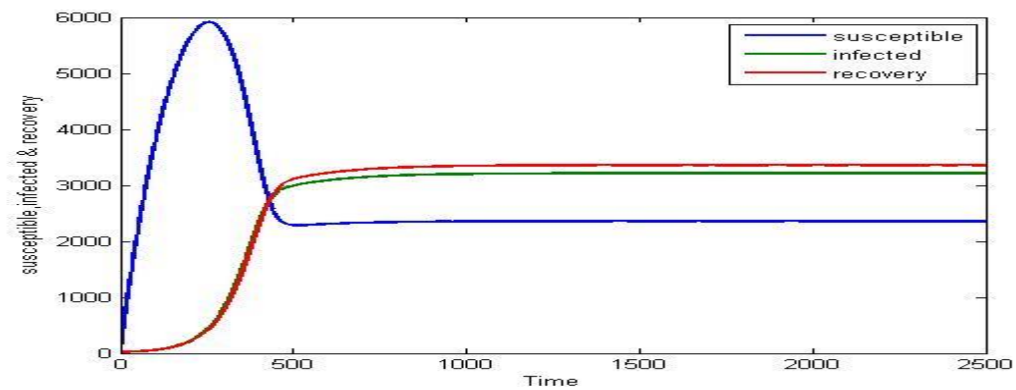

Fig.7

6. STOCHASTIC ANALYSIS

The stochastic version of the model (1) was formulated in this section to take the effect of noise into account. Population variance intensities (fluctuations) around the positive point of equilibrium due to noise have been measured. The random noise that is applied to the model in the form of additive Gaussian white noise and the disturbances are as follows:

$$\frac{dS}{dt} = b - \mu S - \frac{\beta IS}{S + \rho I} + \delta R + p_1 \xi_1(t)$$
$$\frac{dI}{dt} = \frac{\beta IS}{S + \rho I} - \mu I - \delta I - \frac{\alpha I}{1 + \delta I} + p_2 \xi_2(t)$$
$$\frac{dR}{dt} = \gamma I - \mu R - \delta R + \frac{\alpha I}{1 + \delta I} + p_2 \xi_3(t)$$

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Where $p_{1,} p_2, p_3$ real constants are $\xi(t) = (\xi_1(t), \xi_2(t), \xi_3(t))$ is a 3D Gaussian White noise process satisfying $E[\xi_i(t)] = 0; i = 1, 2, 3$

$E[\xi_i(t)\xi_j(t')] = \delta_{ij}\delta(t - t'); i = j = 1, 2, 3$ Where δ_{ij} the Kronecker is symbol; δ is the δ-Dirac function.

Let $S(t) = \mu_1 + \overset{*}{S}; I(t) = \mu_2 + \overset{*}{I}; R(t) = \mu_3 + \overset{*}{R};$

$$\frac{dS}{dt} = \frac{du_1}{dt}; \frac{dI}{dt} = \frac{du_2}{dt}; \frac{dR}{dt} = \frac{du_3}{dt};$$

Linear parts of above equations are

$$\frac{du_1}{dt} = p_1\xi_1(t), \frac{du_2}{dt} = 2\alpha\delta u_2 \overset{*}{I} + p_2\xi_2(t) \,\&\, \frac{du_3}{dt} = p_3\xi_3(t)$$

Taking F.T on both sides of above equations, we get

$$p_1 \bar{\xi}_1(\omega) = i\omega \bar{u}_1(\omega),\ p_2 \bar{\xi}_2(\omega) = \left(i\omega - 2\alpha\delta \overset{*}{I}\right).\bar{u}_2(\omega),\ p_3 \bar{\xi}_3(\omega) = i\omega.\bar{u}_3(\omega)$$

The matrix forms of above equations are $M(\omega)\bar{u}(\omega) = \bar{\xi}(\omega)$

When $$M(\omega) = \begin{bmatrix} i\omega & 0 & 0 \\ 0 & i\omega - 2\alpha\delta \overset{*}{I} & 0 \\ 0 & 0 & i\omega \end{bmatrix}; \bar{u}(\omega) = \begin{bmatrix} \bar{u}_1(\omega) \\ \bar{u}_2(\omega) \\ \bar{u}_3(\omega) \end{bmatrix}; \bar{\xi}(\omega) = \begin{bmatrix} p_1 \bar{\xi}(\omega) \\ p_2 \bar{\xi}(\omega) \\ p_3 \bar{\xi}(\omega) \end{bmatrix};$$

Above form can be written as $\bar{u}(\omega) = [M(\omega)]^{-1} \bar{\xi}(\omega)$

Let $[M(\omega)]^{-1} = K(\omega)$

Therefore, $\bar{u}(\omega) = K(\omega)\bar{\xi}(\omega)$ Where $K(\omega) = \dfrac{Ads(M(\omega))}{|M(\omega)|}$

If the function Y(t) has a zero mean value, Then the fluctuation intensity(variance) of its components in the frequency intervals $(\omega, \omega + d\omega)$ is $S_y(\omega)d\omega$. Where $S_y(\omega)$ is spectral density of Y and is defined as $$S_y(\omega) = \underset{\bar{T} \to +\infty}{Lt} \frac{\left|\bar{y}(\omega)\right|^2}{\bar{T}}$$

If Y has a zero mean value, the invertible transform of $S_y(\omega)$ is the auto covariance function

$$C_y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_y(\omega)(e)^{i\omega\tau}\, d\omega$$

The corresponding variance of fluctuations in y(t) is given by

$$\sigma_y^2 = C_y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_y(\omega)d\omega$$

The intensities of fluctuations in the variables u_i's; $i = 1, 2, 3$ are given by

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$$\sigma_{u_i}^2 = \frac{1}{2\pi}\sum_{j=1}^{3}\left\{\int_{-\infty}^{\infty}\alpha_j \left|K_{ij}(\omega)\right|^2 d\omega\right\}; i, j = 1,2,3$$

From, we obtain

$$\sigma_{u_1}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} p_1 \left|\frac{A_1}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_2 \left|\frac{B_1}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_3 \left|\frac{C_1}{\left|M(\omega)\right|}\right|^2 d\omega\right\}$$

$$\sigma_{u_2}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} p_1 \left|\frac{A_2}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_2 \left|\frac{B_2}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_3 \left|\frac{C_2}{\left|M(\omega)\right|}\right|^2 d\omega\right\}$$

$$\sigma_{u_3}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} p_1 \left|\frac{A_3}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_2 \left|\frac{B_2}{\left|M(\omega)\right|}\right|^2 d\omega + \int_{-\infty}^{\infty} p_3 \left|\frac{C_3}{\left|M(\omega)\right|}\right|^2 d\omega\right\}$$

$$where \in \left|M(\omega)\right| = \left[2\alpha\delta\omega^2 \overset{*}{I}\right] + i\left[-\omega^3\right] = R(\omega) + i\left(\operatorname{Im} g(\omega)\right)$$

$$\left|A_1(\omega)\right|^2 = \left(4\alpha^2\delta^2\omega^2 \overset{*}{I}^2 + \omega^2\right); \left|B_1(\omega)\right|^2 = 0; \left|C_1(\omega)\right|^2 = 0; \left|A_2(\omega)\right|^2 = 0; \left|B_2(\omega)\right|^2 = \left(\omega^2\right); \left|C_2(\omega)\right|^2 = 0;$$

$$\left|A_3(\omega)\right|^2 = 0; \left|B_3(\omega)\right|^2 = 0; \left|C_3(\omega)\right|^2 = \left(\omega^2\right) + 4\alpha^2\delta^2\omega^2 \overset{*}{I}^2$$

Therefore

$$\sigma_{u_1}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} \frac{\alpha_1\left(\omega^2 + 4\omega^2\alpha^2\delta^2 \overset{*}{I}^2\right)}{\omega^6 + 4\omega^4\alpha^2\delta^2 \overset{*}{I}^2}\right\} d\omega \quad \sigma_{u_2}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} \frac{\alpha_2\left(\omega^2\right)}{\omega^6 + 4\omega^4\alpha^2\delta^2 \overset{*}{I}^2}\right\} d\omega$$

$$\sigma_{u_3}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} \frac{\alpha_3\left(\omega^2 + 4\omega^2\alpha^2\delta^2 \overset{*}{I}^2\right)}{\omega^6 + 4\omega^4\alpha^2\delta^2 \overset{*}{I}^2}\right\} d\omega$$

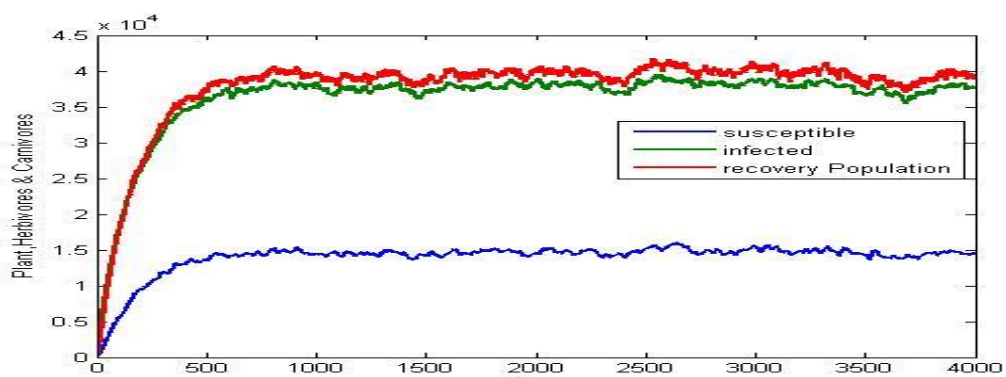

Fig.8

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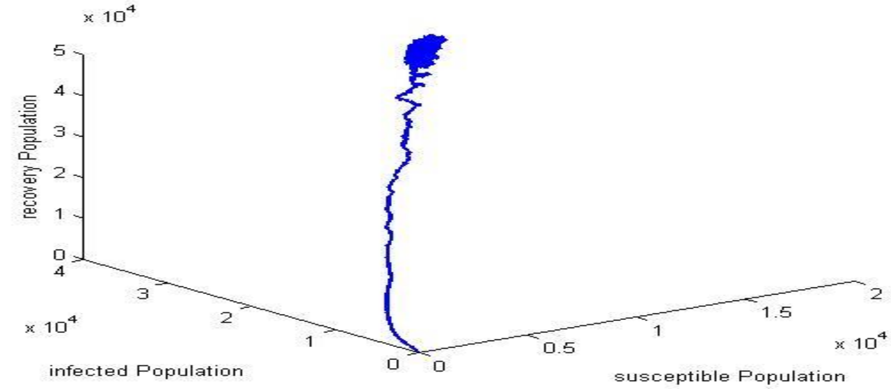

Fig.9

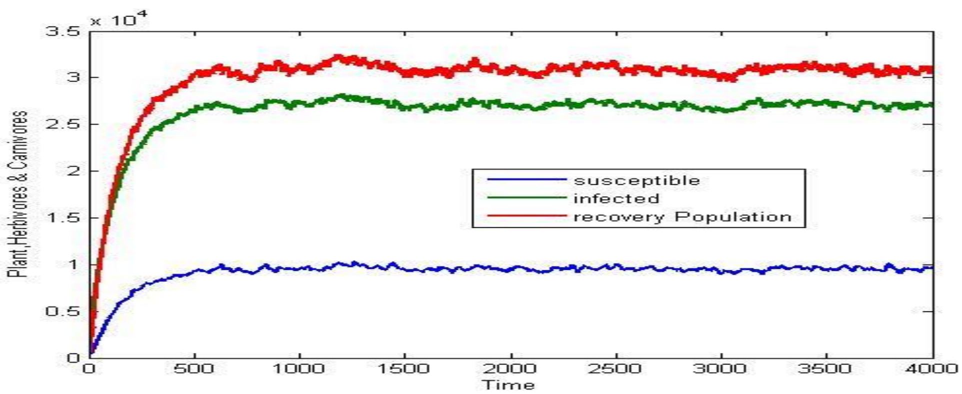

Fig.10

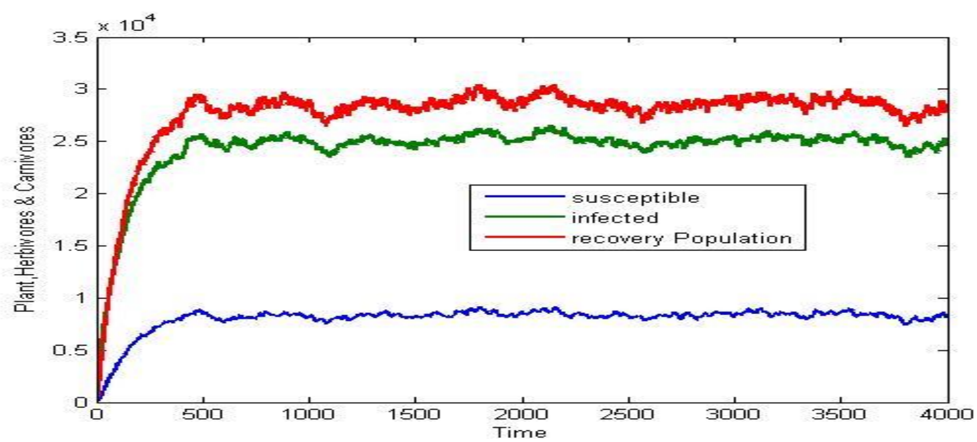

Fig.11

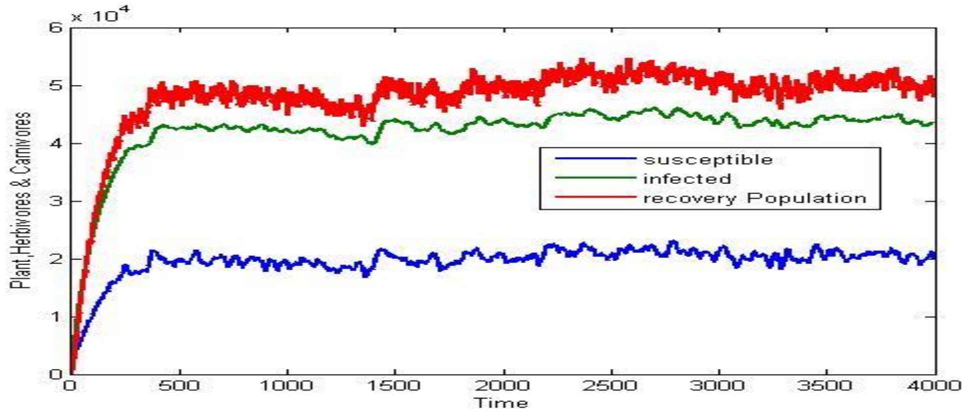

Fig.12

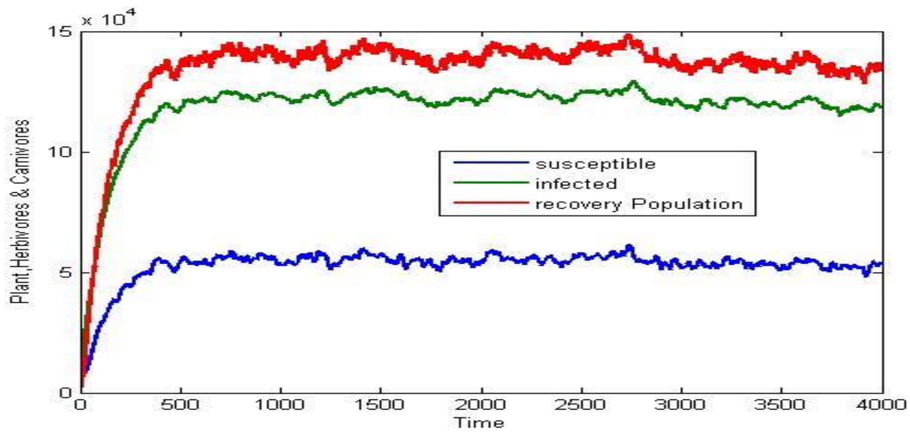

Fig.13

7. CONCLUSION

Based on the investigation on this SIR an epidemic model, the number R_0 is derivedthe point E_0 is asymptotically locally and globally stable when $R_0 < 1$ and unstable if $R_0 > 1$. A unique E^* exits when $R_0 = \frac{\beta}{\mu + \gamma + \alpha} < 1$. By using computer simulation, the dynamic behavior of interacting populations is investigated. Increases in the transfer rate have been noted, with the disease being more endemic. To avoid the spread of the disease, the population should make an effort to decrease the rate of transmission. It is noted in Fig 6 that when the treatment is very bad, the invectives are very high and infectives decrease when the treatment improves. It is noted that the saturation term for the infectives is increased to increase the susceptible and recovered individuals. This shows that the saturation principle has a positive effect on the eradication of diseases.By integrating the Gaussian White noise under the influence of fluctuating conditions, we have formulated the stochastic version of the model. The periodic conduct of population growth in a random setting of high and low intensity is seen in figures (8) to (13).The inclusion of stochastic disturbance is observed according to the above analysis, which results in a significant change in the strength of our model system due to a small change in sensitive parameters that causes large fluctuations in the environment.

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