Dominating Set of Fuzzy and Anti-Fuzzy Graphs Using Algorithms

Nageswara Rao Thota[1, a)], Muneera Abdul[1, b)], Y.V. Seshagiri Rao[2, c)]

E.S.Rama Ravi Kumar[3, d)], JonnalagaddaVenkateswaraRao [4,e)]

[1] *Department of Engineering Mathematics,* College of Engineering, *Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.*

[2]*Department of Mathematics, Vignan Institute of Technology and Science, Hyderabad, Telangana, India.*

[3]*Department of Mathematics, V.R.Siddhartha Engineering College, Vijayawada, Andhra Pradesh, India.*

[4]*Department of Mathematics, School of Science & Technology,United States International University, USIU-Africa, P.O. Box 14634 - 00800 Nairobi, Kenya.*

[a)] Corresponding author: tnraothota@kluniversity.in

[b)]munny.aliet@gmail.com

[c)] yangalav@gmail.com

[d)] srrkemani1@gmail.com

[e)] jvrao@usiu.ac.ke

Abstract: In this article, we discuss a dominating set D in a fuzzy graph F and anti fuzzy graph F_A. The boundaries on the domination number of an anti-fuzzy graph are obtained. A fuzzy matrix is defined on an anti-fuzzy graph. Some properties of the adjacency fuzzy matrix are discussed. Using a strong adjacency matrix, an algorithm is formulated to find a minimal dominating set of an anti fuzzy graph F_A and by using a strong neighborhood fuzzy matrix, an algorithm is defined to predict the total dominating set for F_A.

INTRODUCTION

Uncertainty has a pivotal role in any efforts to maximize the usefulness of system models. In real-world problems, the conditions are very often uncertain or vague in several ways. Due to a lack of information, the upcoming state of the system might not be known completely. Fuzziness is the uncertainty subsequent from inaccuracy of meaning of an idea expressed by a linguistic term in Natural languages, such as “tall” or “warm” etc. Fuzzy logic provides a systematic basis for the depiction of imprecision, uncertainty, vagueness, and/or incompleteness. By crisp we mean yes or no type rather than more or less type.

PRELIMINARIES

Definition: A fuzzy graph F (V, σ, μ) is a pair of functions, the vertex set $\sigma: V \to [0, 1]$ is a fuzzy subset of V, and edge set $\mu: V \times V \to [0, 1]$ is a fuzzy relation on V such that $\mu(x, y) \le \min\{\sigma(x), \sigma(y)\}$ for every x, y ϵ V.

Ex: F (V, σ, μ) is a fuzzy graph with V = {a, b, c}. σ (a) =1, σ (b) = 0.9, σ (c) = 0.4, μ (a, b) = 0.7, μ (a, c) = 0.3, μ (b, c) = 0.2.

Definition: Let F (V, σ, μ) be a fuzzy graph. $D \subseteq V$ is said to be fuzzy dominating set of F (V, σ, μ), if for each and every node x ϵ V – D, ∃ a node y ϵ D, such that y dominates x. (or) A subset D of V is said to be a dominating set of F if for every x ϵ V – D there exists an element y ϵ D such that $\mu(y, x) = \sigma(y) \wedge \sigma(x)$.

Note: Every superset of a fuzzy dominating set is a fuzzy dominating set, but a subset of a fuzzy dominating set need not be a fuzzy dominating set.

1st International Conference on Essence of Mathematics and Engineering Applications
AIP Conf. Proc. 2707, 020034-1–020034-6; https://doi.org/10.1063/5.0143097
Published by AIP Publishing. 978-0-7354-4516-1/$30.00

Definition: A fuzzy dominating set D of a fuzzy graph F (V, σ, μ), is called minimal fuzzy dominating set, if, for every node v ϵ D, D – {v} is not a fuzzy dominating set. Minimum cardinality among all minimal fuzzy dominating sets of F (V, σ, μ) is called a fuzzy domination number of F and is represented by γ(F).

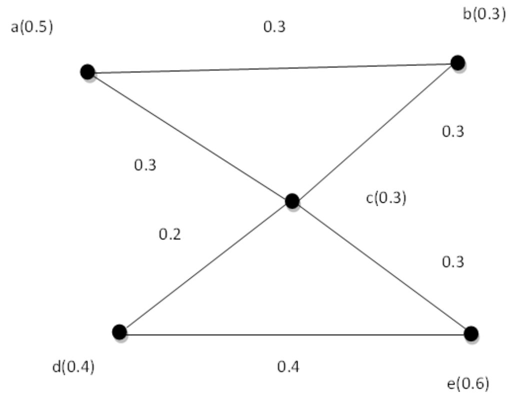


Fig.1: Minimal dominating set

In the above Fig.1 fuzzy graph D = {a, e} is a the minimal dominating set & the domination number if γ (F) = 1.1

<u>**An Algorithm to find Dominating Set**</u>

Step 1: Find μ^{∞} (x, y) for all edges (x, y)

Step 2: Remove all the weak arcs (or) edges

Step 3: Choose the vertex x with maximum σ (x) in F'

Step 4: Group the nodes dominated by x, as V_1

Step 5: Find F' - V_1

Step 6: Repeat the 3 to 5 steps till we get isolated nodes.

Step 7: The collection of nodes chosen in step 3 and the isolated nodes will form a dominating set.

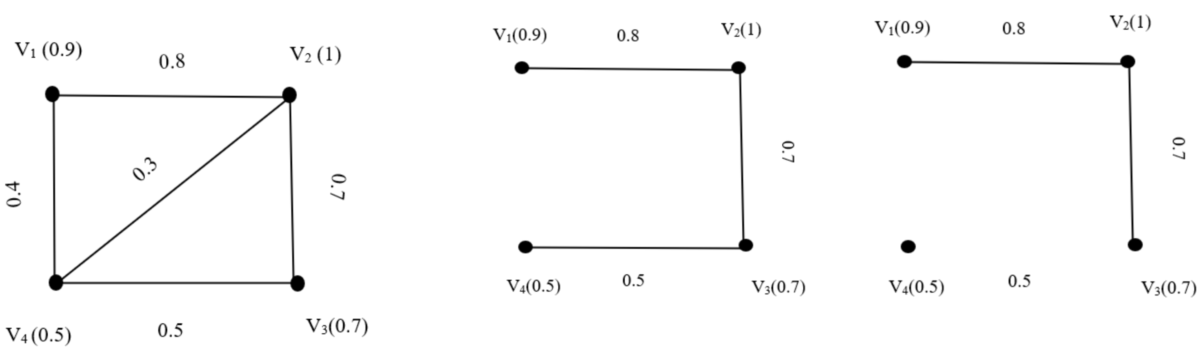


Fig.2 Example of Dominating set

From Fig.2,The Dominating set D = {V_2, V_4}

Definition: Let the fuzzy graph F (V, σ, μ) is said to be an antifuzzy graph with a vertex set σ: V → [0, 1] and edge relation μ: V × V → [0, 1], where ∀ x, y ∈ V, where μ (x, y) ≥ max{σ(x), σ(y)} and it is represented by F_A (σ, μ).

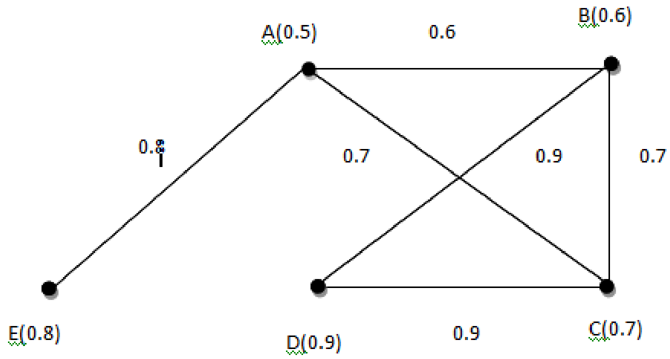


Fig.3: Example Figure

In the above Fig.3.Degree of a node C is σ(C) = μ (A, C) + μ (B, C) + μ (D, C)

= 0.7 + 0.7 + 0.9

= 2.3.

Here d (σ(A)) = 2.1, d(σ(B)) = 2.2, d(σ(C)) = 2.3, d(σ(D)) = 1.8, d(σ(E)) = 0.8

Definition: Let F_A be an anti fuzzy graph and x, y ∈ V (F_A). If y is said to be a support node to x then y is adjacent to at least one end node x in F_A.

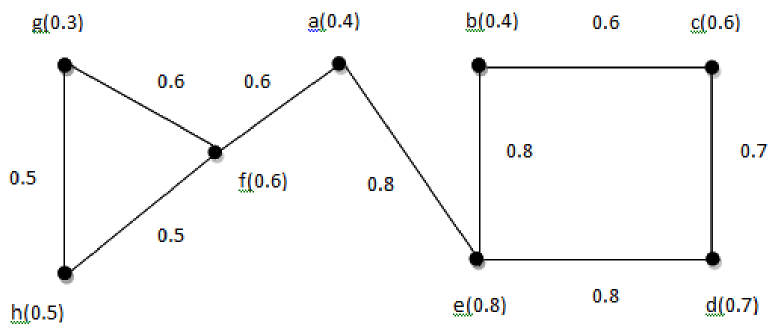


Fig.4: Example for total Dominating sets

In the above Fig.4, Total Dominating sets are {d, e, f, h}, {d, e, f, g} & {d, e, f, a}.

Corresponding total dominating numbers are 2.3,2.6 & 2.4 respectively.

Hence, {d, e, f, g}is a minimal Total Dominating set of F_A and $\gamma_t\ (F_A)$ = 2.6

Theorem: For a linked anti-fuzzy graph F_A, with m ≥ 2 then $\gamma(F_A) \geq \frac{p}{1+\Delta(F_A)}$

Proof: Let us Consider a minimal dominating set D with maximum fuzzy value of anti-fuzzy graph F_A then $|D|_f$. $\Delta(F_A) \geq |v \backslash D|_f$

⇛ $|D|_f$. $\Delta(F_A) \geq \sum_{u \in V} \sigma(u) - |D_f|$

⇛ $\gamma(F_A)$. $\Delta(F_A) \geq p - \gamma$

⇛ $\gamma(F_A)$. $\Delta(F_A) + \gamma(F_A) \geq p$

$\Rightarrow \gamma(F_A)\,[\Delta(F_A)+1] \geq p$

$\Rightarrow \gamma(F_A) \geq \frac{p}{1+\Delta(F_A)}$

Algorithm

Algorithm to determine Dominating set of Antifuzzy Graph

Definition: An antifuzzy graph F_A (σ, μ) with μ to be reflexive & symmetric is determined by the adjacency fuzzy matrix and it is denoted by M_μ.

$$(M_\mu)_{i,j} = \begin{cases} \mu(v_{i\,,}\, v_i)\; for\; i\; \neq j \\ \sigma(v_i),\;\; for\; i\; = j \end{cases}$$ and M_μ is a square matrix

Definition: Let F_A be a connected antifuzzy graph. The strong adjacency matrix $(M_\mu)'$ is

$$(M_\mu)' == \begin{cases} \mu(v_{i\,,}\, v_i)\; for\; i\; \neq j \\ \sigma(v_i) \end{cases}$$ and v_i is a strong neighbour to v_j.

Algorithm

1 Step: For F_A, construct $(M_\mu)'$

2 Step: Consider the Dominating set D = ∅

3 Step: Find $\sum R\ (u_i)$ and $\sum C\ (u_i)$

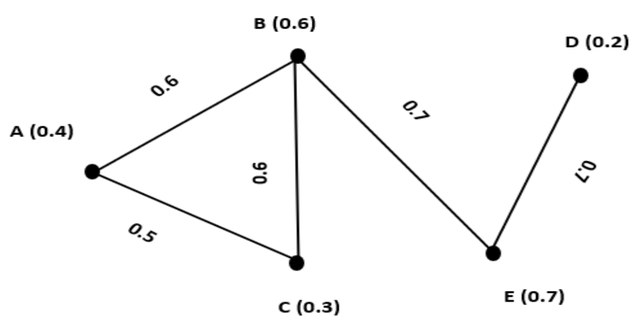
4 Step: In $(M_\mu)'$, yield the maximum value in R (u_i). Draw happens then select the node with maximum $\sigma(u_i)$ in main diagonal or else break it randomly.

5 Step : Pick-up the conforming node in the designated row as a dominated node. And consider the node in the dominating set D.

6 Step : Cut the particular row with horizontal line. By using vertical lines, cover all the entries in column wise which conforming to the particular row. The subsequent reduced matrix is defined as $(M_\mu)'_1$.

7 Step: If the subsequent condensed matrix $(M_\mu)'_1$ has any rows go to 3step. Otherwise STOP.

8 Step: Lastly get the Dominating set D of F_A, and it will be minimal.



e

Fig.5: Example Figure

1Step: From Fig.5

$$(M_\mu)' = \begin{bmatrix} 0.4 & 0.6 & 0 & 0 & 0 \\ 0.6 & 0.6 & 0.6 & 0 & 0.7 \\ 0 & 0.6 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.7 \\ 0 & 0.7 & 0 & 0.7 & 0.7 \end{bmatrix}$$

2Step: Consider dominating set D = ∅

3Step: $\sum R(u_i)$

$$(M_\mu)' = \begin{bmatrix} 0.4 & 0.6 & 0 & 0 & 0 \\ 0.6 & 0.6 & 0.6 & 0 & 0.7 \\ 0 & 0.6 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.7 \\ 0 & 0.7 & 0 & 0.7 & 0.7 \end{bmatrix} \quad \begin{matrix} 1 \\ 2.5 \\ 0.9 \\ 0.9 \\ 2.1 \end{matrix}$$

$\sum C(u_i)$ 1 2.5 0.9 0.9 2.1

4Step: In $(M_\mu)'$, Total has maximum value in the second row. So, select B. Hence D = {B}

5Step: Remove the nominated row and the agreeing column A, C, and E in $(M_\mu)'$.

Then reduced ensuing matrix is $(M_\mu)_1'$.

6Step: $(M_\mu)_1' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \\ 0.7 \end{bmatrix}$

7Step: $(M_\mu)_1'$ has only one row. We stop.

We get the Dominating set as D = {B, D} for antifuzzy graph F_A.

Hence Domination Number $\gamma = 0.8$

CONCLUSION

Fuzzy graph theory is tremendously applied in software engineering applications, in this article we explicit some sorts of Algorithms. Algorithm applied to discovery the dominating set for a assumed fuzzy graph and row maxima Algorithm is characterized to define the minimal dominating set for a chosen antifuzzy graph.

REFERENCES

1. L.A.Zadeh, Fuzzy sets, Inform.control,8(1965) 338-353.
2. A. Nagoorgani and K. Radha, Regular Property of Fuzzy Graphs, Bulletin of Pure and Applied Sciences, Volume 27E, Number 2, 2008, 411-419.
3. M.S. Sunitha and Sunil Mathew, Types of arcs in a fuzzy graph, Information Sciences, 179 (2009) 1760 -1768.
4. Bhutani KR, Rosenfeld A (2003) Fuzzy end nodes in fuzzy graphs. Inf Sci 152: 323 -326.
5. J.N. Moderson and P.S. Nair, Cycles and Cocycles of fuzzy graphs, Information Sciences, 79 (1994), 169-170.
6. J.N. Mordeson, C.S. Peng, Operations on fuzzy graphs, Information Sciences 79 (1994) 159-170.
7. C. Pang; R. Zhang, Q. Zhang, J.Wang, Domination sets in directed graphs, Information Sciences 180 (2010) 3647-3652.

8. Bhutani KR, Rosenfeld A (2003) Strong arcs in fuzzy graphs. Inf Sci 152: 319-322.
9. Nageswar Rao. T, Muneera Abdul, Jonnalagadda Venkateswara Rao, M. N. Srinivas, Sateesh Kumar. D., Trends on Dominations of Fuzzy Graphs and Anti Fuzzy Graphs, AIP Conference Proceedings 2375, (2021), DOI https://doi.org/10.1063/5.0066458 pp: 020013.
10. Bhagavan, V. S., Tadikonda, S., & Sateesh Kumar, D. (2021). A class of generating functions for chebshev polynomials by weisner method. Paper presented at the AIP Conference Proceedings, , 2375 doi:10.1063/5.0066695
11. Lakshmi, K., Mahaboob, B., Sateesh Kumar, D., Balagi Prakash, G., & Nageswara Rao, T. (2021). A new vision on ordinary least squares estimation of parameters of linear model. Paper presented at the AIP Conference Proceedings, , 2375 doi:10.1063/5.0066922
12. Mahaboob, B., Praveen, J. P., Prasad, V. B. V. N., Rao, T. N., Prakash, G. B., & Kumar, D. S. (2021). Lexi-search algorithm using pattern recognition technique for solving a variant constrained bulk transshipment problem. Journal of Mathematical and Computational Science, 11(3), 3436-3463. doi:10.28919/jmcs/5679