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RADIATION EFFECT ON UNSTEADY FREE CONVECTION OSCILLATORY COUETTE FLOW THROUGH A POROUS MEDIUM WITH PEROIDIC WALL TEMPERATURE AND HEAT GENERATION

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.***Abstract:*** The present paper analyzes the radiation effects on the heat transfer and magnetohydrodynamic free convection flow through a highly porous plate. It is assume that free stream velocity oscillates in times about a constant mean. Assume that periodic temperature at the moving plate. The momentum equation and energy equation which govern the flow field are solved by regular perturbation technique. Analytical solutions are derived for transient velocity, transient temperature profiles and tangent of phase skin-friction. The behavior of the mean velocity, mean temperature and Nusselt number has been discussed for variations in the physical parameters with the help of graphs and tables. The exactitude of the problem has been certified by matching with the previous available work and the agreement between the results is outstanding, which established confidence in the numerical results described in this article.

Keywords: Radiation, Coutte flow, free convection, MHD, Heat generation.

1. INTRODUCTION

Fluid dynamics is the science, which deals with the properties of fluids in motion. The subject dealing with motion of electrically conducting fluids in the presence of magnetic field is named as MHD (magnetohydrodynamic). 'Continuum Hypothesis' and 'Newtonian Mechanics' is goes to behave as the source for study of fluid dynamics. Every single physical system in nature is subjected to small perturbations. If the system is disturbed and the disturbances gradually die down, then the system is said to be stable. If the perturbations increases with time that is, the system no way reverts to its initial position, is called unstable. If the system neither departs from its disturbed state nor tends to return to initial position, the system is called in neutral equilibrium. Further, if there's an oscillatory motion with growing amplitude, the instability is named as over stability. Instability of the system even for a single mode of disturbance will qualify the system to be unstable, whereas the system can't be considered as the stable unless it is stable with respect to every possible disturbance to which it can be subjected upon. The linearized stability theory and normal mode analysis have been used throughout this study. Boussinesq's approximation has been used, because, in solving the hydrodynamic equations we have difficulties regarding their non-linear character and the variable nature of various coefficients due to the variation in temperature. In view of the above some of the authors shown interest Nield (1994) investigated convection in a porous medium with inclined temperature gradient: An additional results. The authors Karunakar Reddy et al. (2013) studied MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction. Martynenko et al. (1984) discussed laminar free convection from a vertical plate. The authors (Srinathuni Lavanya and Chenna Kesavaiah 2017) investigated heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction. The authors (Chenna Kesavaiah and Sudhakaraiah 2014) investigated effects of heat and mass flux to MHD flow in vertical surface with radiation absorption. Harris et al. (2014) studied free convection from a vertical plate in porous medium subjected to a sudden change in surface temperature. A.Neelima and Omesshwar Reddy.V (2021) considered the effects of Unsteady MHD Oscillatory Flow of a Viscous Stratified Fluid Past a Porous Flat Moving Plate. The authors (Bhavana and Chenna Kesavaiah 2018) investigated perturbation solution for thermal diffusion and chemical reaction effects on MHD flow in vertical surface with heat generation.

In the recent years there has been a renewed interest in studying MHD (magneto-hydrodynamics) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow has attracted the interest of many investigators in view of its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Chenna Kesavaiah et al. (2013) investigated radiation and thermo-diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source. Omeshwar Reddy et al. (2019) discussed the Finite difference solutions of MHD natural convection visco elastic fluid flow past a vertically inclined porous plate in presence of thermal diffusion, diffusion thermo, heat and mass transfer effects. Das et al. (1999) studied transient free convection flow past an infinite vertical plate with periodic temperature variation. Pawan kumar sharma et al. (2007) investigated unsteady free convection oscillatory coquette flow through a porous medium with periodic wall temperature. Same related investigations found by some authors Chenna Kesavaiah et al. (2013), Ganesan et al. (1999), Singh (2000), D Ch Kesavaiah et al. (2012), Cogly A C et al. (1968).

The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Now, free convective flow past a vertical plate has been studied extensively by Chenna Kesavaiah et al. (2013) discussed natural convection heat transfer oscillatory flow of an elastic-viscous fluid from vertical plate. The author (Narahari 2013) investigated effects of thermal radiation and free convection currents on the unsteady Coutte flow between two vertical parallel plates with constant heat flux at one boundary. Ch Kesavaiah et al. (2011) discussed effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. The authors (Omeswar Reddy and V Thiagarajan 2019) analyzed the visco elastic fluid effect on mixed convective fluid flow past a way inclined porous plate in presence of Soret, Dufour, heat source and thermal radiation. Chenna Kesavaiah and Devika (2020) has been studied free convection and heat transfer of a Coutte flow an infinite porous plate in the presence radiation effect. The authors (Chenna Kesavaiah and Venkateswarlu 2020) considered chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves. Mallikarjuna Reddy et al. (2019) analyzed radiation and diffusion thermo effects of viscoelastic fluid past a porous surface in the presence of magnetic field and chemical reaction with heat source. The author (Chenna Kesavaiah 2021) analyzed radiative MHD Walter's Liquid-B flow past a semi-infinite vertical plate in the presence of viscous dissipation with a heat source. Rami Reddy et al. (2021) explained Hall Effect on

MHD flow of a visco-elastic fluid through porous medium over an infinite porous plate with source. Chenna Kesavaiah and Venkateswarlu (2020) studied chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves. Nagaraju et al. (2020) investigated radiation and chemical reaction effects on MHD casson fluid flow of a porous medium with suction/injection. Some related investigations are found in Sanatan Das et al. (2012), H. M. Joshi (1988), A. Ogulu and S. Motsa (2005), B. K. Jha (2001), M. R. Abdullah and N. Saada (2019), N. Ahmed et al. (2012), K. Mandal et al. (2014), B. Zigta and P. Koya (2017), J.K. Singh (2016)

The objective of the present chapter is to analyze the radiation effects on an unsteady magnetohydrodynamic free convection oscillatory Coutte flow through a highly porous medium when the temperature of the plate oscillates in time. The momentum equation and energy equation which govern the flow field are solved by regular perturbation technique. Analytical solutions are derived for transient velocity, transient temperature profiles and tangent of phase skin-friction. The behavior of the mean velocity, mean temperature and Nusselt number has been discussed for variations in the physical parameters with the help of graphs.

2. Formulation of the Problem

An unsteady Couette flow of a viscous incompressible fluid through a highly porous medium and bounded between two infinite vertical porous plates in the presence of radiation, heat generation/ absorption is considered. One of which is suddenly moved from rest with a free stream velocity that oscillate in time about a constant mean. Further, it is assumed that the temperature of moving plate fluctuates in time about a non-zero constant mean. We take $x^* -$ axis along the moving vertical plate in the vertically upward direction and $y^* -$ axis is taken normal to this plate. The other stationary vertical plate is assumed to be situated at $y^* = b$ at temperature T_s^*. We consider the free-stream velocity distribution of the form:

$$U^*\left(t^*\right) = U_0\left(1 + \varepsilon e^{i\omega^* t^*}\right) \tag{2.1}$$

where $U_0 -$ is the mean constant free-stream velocity, $\omega^* -$ is the frequency of oscillations and $t^* -$ is the time. The equations governing the problem are:

Momentum Equation

$$\rho \frac{\partial u^*}{\partial t^*} = -\frac{\partial P}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \rho g - \frac{\mu}{k^*} u^* - \sigma B_0^2 u^* \tag{2.2}$$

Equation of Energy:

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\left(\frac{\partial q^*}{\partial y^*}\right) - \frac{Q_0}{\rho C_p}\left(T^* - T_s^*\right)$$

(2.3)

Boundary conditions

$$y^* = 0 : u^* = U_0\left(1+\varepsilon e^{i\omega^* t^*}\right), T^* = T_n^* + \varepsilon\left(T_n^* - T_s^*\right)e^{i\omega^* t^*}$$
$$y^* = b : u^* = 0, T_n^* = T_s^* \tag{2.4}$$

where $u^*, U^*, \rho, \mu, P, g, \beta, k^*, \alpha, T_n^*, T_s^*$ s are respectively, velocity, free-stream velocity, density, viscosity, pressure, gravity, volumetric coefficient of thermal expansion, permeability parameter, thermal diffusivity, temperature of fluid in the boundary layer, temperature of the moving plate and temperature of the stationary plate.

The $(*)$ stands for dimensional quantities.

The radiative heat flux is given by Cogly (1968)

$$\frac{\partial q_r^*}{\partial y^*} = 4\alpha^2(T^* - T_s^*)$$

where $\alpha^2 = \int_0^\infty \delta\lambda \frac{\partial B}{\partial T^*}$, $B-$ is Planck's function.

Equation (2), for the free stream, is reduced to

$$\rho\frac{dU^*}{dt^*} = -\frac{\partial^2 P}{\partial x^*} - g\rho_\infty - \frac{U^*\mu}{k^*} - \sigma B_o^2 U^* \tag{2.5}$$

From equations (2) and (5), we get

$$\rho\frac{\partial u^*}{\partial t^*} = \rho\frac{dU^*}{dt^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - g\left(\rho_\infty - \rho\right) - \frac{\left(u^* - U^*\right)\mu}{k^*} - \sigma B_o^2\left(u^* - U^*\right)$$

(2.6)

This equation is reduced to

$$\rho\frac{\partial u^*}{\partial t^*} = \rho\frac{dU^*}{dt^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \rho g\beta\left(T^* - T_s^*\right) - \frac{\left(u^* - U^*\right)\mu}{k^*} - \sigma B_o^2\left(u^* - U^*\right)$$

(2.7)

by using the constitutive equation

$$g\left(\rho_\infty^* - \rho^*\right) = \rho g\beta\left(T^* - T_s^*\right)$$

where $\beta-$ is the volumetric coefficient of thermal expansion and ∞ - the density of the fluid far away the surface.

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{b}, u = \frac{u^*}{U_0}, U = \frac{U^*}{U_0}, t = \omega^* t^*, \theta = \frac{T^* - T_s^*}{T_n^* - T_s^*}, k = \frac{k^*}{b^2}, M^2 = \frac{\sigma B_0^2 b^2}{\rho\nu}$$
$$Gr = \frac{g\beta b^2\left(T_n^* - T_s^*\right)}{\nu U_0}, \quad R = \frac{4\alpha b^2}{\rho Cp}, \quad \Pr = \frac{\nu}{\alpha}, \quad \phi = \frac{Q_0 b^2}{\nu\rho Cp}, \omega = \frac{\omega^* b^2}{\nu} \tag{2.8}$$

where Gr - Grashof number, $R-$ Radiation parameter, $\Pr-$ Prandtl number, $M-$ Magnetic parameter, $\phi-$ Heat source parameter

Introducing the radiative heat flux in equation (2.3), then the equations (2.3) and (2.6) become

$$\omega\frac{\partial u}{\partial t}=\omega\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}-Gr\theta-\left(\frac{1}{K}+M^2\right)(u-U) \tag{2.9}$$

$$\omega\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}-R\theta-\Pr\phi\theta \tag{2.10}$$

with corresponding boundary conditions

$$y=0: u=1+\varepsilon e^{it}, \theta=1+\varepsilon e^{it}$$
$$y=1: u=0, \theta=0 \tag{2.11}$$

1. Solution of The Problem

Since the amplitudes of the free-stream velocity and temperature variation $\varepsilon(<<1)$ is very small, we now assume the solutions of the following form:

$$u(y,t)=u_0(y)+\varepsilon u_1(y)e^{it}$$
$$\theta(y,t)=\theta_0(y)+\varepsilon\theta_1(y)e^{it} \tag{3.1}$$

and free - stream velocity

$$U=1+\varepsilon e^{it} \tag{3.2}$$

Substituting equations (2.10) and (2.11) in equations (2.7) and (2.8), comparing the coefficients of identical power of ε and neglecting those of ε^2, we get following equations

$$u_0''-N_1u_0=-Gr\theta_0-N_1 \tag{3.3}$$

$$\theta_0''-N_2\theta_0=0 \tag{3.4}$$

$$u_1''-\left(i\omega+\frac{1}{k}\right)u_1=-Gr\theta_1-\left(i\omega+\frac{1}{k}\right) \tag{3.5}$$

$$\theta_1''-N_3\theta_1=0 \tag{3.6}$$

where $N_1=\frac{1}{K}+M^2, N_2=(\Pr\phi+R),\ N_3=(N_2+i\omega\Pr)$

with the corresponding boundary conditions:

$$y=0: u_0=1, u_1=1, \theta_0=1, \theta_1=1$$
$$y=1: u_0=0, u_1=0, \theta_0=0, \theta_1=0 \tag{3.7}$$

where primes denotes differentiation with respect to y. solving these equations under the corresponding boundary conditions are

$$\theta_0(y)=B_2e^{m5y}+B_1e^{m6y}$$
$$u_0(y)=A_1e^{m2y}+A_2e^{m1y}+J_1e^{m5y}+J_2e^{m6y}-1$$
$$\theta_1(y)=B_3e^{m8y}+B_4e^{m7y}$$
$$u_1(y)=A_3+A_4+A_5e^{m4y}+A_6e^{m3y}$$
$$\theta(y,t)=B_1e^{m6y}+B_2e^{m5y}+\varepsilon e^{it}\left\{B_3e^{m8y}+B_4e^{m7y}\right\}$$
$$u(y,t)=A_1e^{m1y}+A_2e^{m2y}+\varepsilon e^{it}\left\{A_3+A_4+A_6e^{m3y}+A_5e^{m4y}\right\}$$

4. Result and Discussions

In order to point out the effect of permeability and convection on the velocity, when the moving plate is subjected to oscillating free-stream velocity and fluctuating wall temperature, the following discussions are set out. Numerical calculations are carried out for different values of Gr, Pr, ω and k. The values of Prandtl number are chosen as 0.71 and 7.0 approximately, which represent air and water respectively at $20^{\circ}C$. The values of Gr and K are selected arbitrarily.

(a) Mean flow

The mean flow velocity is given by equation (4.1). This velocity component is presented in **figure (1)** for different values of the radiation parameter. It is observed that an increase in the radiation parameter the mean flow velocity increases. The temperature profiles are calculated form equation (20) and these are shown in **figure (2)** for air (Pr=0.71). The effect of radiation parameter is important in temperature profiles. It is observed that the mean temperature decreases with increasing radiation parameter. Typical variation of transient temperature profiles along the spanwise coordinate y are shown in **figure (3)** for different values of Prandtl number (Pr). The results show that an increase of Prandtl number (Pr) results in a decreasing in thermal boundary layer thickness and more uniform transient temperature distribution across the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore, heat is able to differ away from the heated surface more rapidly than for higher values of Pr. Hence, the boundary layer thicker and the rate of heat transfer is reduced, for gradient have been reduced. The effect of heat generation ϕ on the mean temperature profiles for different values of ϕ shown in **figure (4)**. It is seen form this figure that the mean temperature profiles decreasing with increasing of heat generation parameter ϕ. **Figure (5)** shows the variation of mean flow velocity for different values of K. It is observed from the figure that the mean velocity decreases with an increase in permeability parameter because the porous material offers resistance to the flow. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering. For different values of magnetic field parameter M, the mean flow velocity profiles plotted in **figure (6)**. It is obvious that the effect of increasing values of M parameter the results is increasing mean velocity distribution across the boundary layer because of application of transfer magnetic field will results a restrictive type of force (Lorentze force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The rate of heat transfer (Nu) versus ϕ is plotted in **figure (7)**. It is obvious that the effect of increasing values of R parameter results increases.

Knowing the mean velocity field from the practical point of view, it is important to know that the effects of Grashof number on mean-skin friction. It is given by:

$$\tau^* = \mu\left(\frac{du^*}{dy^*}\right)_{y^*=0}$$

and in non-dimensional it is given by:

$$\tau = \frac{\tau^* b}{\mu U_0} = \left(\frac{\partial u}{\partial y}\right)_{y^*=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y^*=0} + \varepsilon\left(\frac{\partial u_1}{\partial y}\right)_{y^*=0} e^{it}$$

Denoting the mean skin friction by

$$\tau_m = \mu\left(\frac{du_0}{dy}\right)_{y=0} \tag{4.1}$$

Differentiating $u(y,t)$ at $y=0$ and subtitling in equation (4.1), we get

$$\tau_m = A_1 m_2 + A_2 m_1 + J_5 m_2 + J_2 m_6$$

(b) Unsteady flow

The velocity and temperature fields as given by equations (3.3) to (3.6) respectively can be expressed in terms of fluctuating parts as follows

$$u(y,t) = u_0(y) + \varepsilon e^{it}(M_r + iM_i) \tag{4.2}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{it}(T_r + iT_i) \tag{4.3}$$

where

$$M_r + iM_i = u_1(y) \quad and \quad T_r + iT_i = \theta_1(y) \tag{4.4}$$

We can now write expressions for transient velocity and transient temperature from (4.4) for $t = \frac{\pi}{2}$, as follows

$$u = \left(y, \frac{\pi}{2}\right) = u_0(y) - \varepsilon M_i \tag{4.5}$$

$$\theta = \left(y, \frac{\pi}{2}\right) = \theta_0(y) - \varepsilon T_i \tag{4.6}$$

From equations (20) and (21) we have

$$\tau = A_1 m_2 + A_2 m_1 + J_5 m_2 + J_2 m_6 + \varepsilon e^{it}\left[(m_4 A_5 + A_6 m_3)\right] \tag{4.7}$$

We can express equation (4.7) in terms of the amplitude and phase of skin-friction as

$$\tau = \tau_m + \varepsilon|M|\cos(t+\phi) \tag{4.8}$$

where

$$M = M_r + iM_i = \text{coefficients of } \varepsilon e^{it} \text{ in equation (4.8)}$$

$$|M| = \sqrt{M_r^{\,2} + iM_i^{\,2}}\,;\text{and } \tan\psi \;=\; \frac{M_i}{M_r}$$

The numerical values of |M| are presented in Table 2. It is observed from this table that |M| increases with increasing ω for water while reverse effect is observed for air (for same values of Gr. The increase in Gr leads to increase in amplitude of skin-friction.

We now study the effect of ω on the rate of heat transfer. The rate of heat transfer in terms of the Nusselt number can be obtained as

$$N_u = \frac{q_w^{\,*} b}{k\left(T_n^{\,*} - T_s^{\,*}\right)} = \left(\frac{\partial\theta}{\partial y}\right)_{y^*=0} = \left(\frac{\partial\theta_0}{\partial y}\right)_{y^*=0} + \varepsilon\left(\frac{\partial\theta_1}{\partial y}\right)_{y^*=0} e^{it}$$

$$N_u = m_6 B_1 + m_5 B_2 + \varepsilon e^{it}\left\{m_8 B_3 + m_7 B_4\right\} \tag{4.9}$$

We can express (27) in terms of amplitude and phase of heat transfer as

$$N_u = -1 + \varepsilon |H| \cos(t + \psi) \qquad (4.10)$$

where

$H = H_r + iH_i =$ coefficients of εe^{it} in equation (4.9)

$$|H| = \sqrt{H_r^2 + iH_i^2}; \text{and } \tan\psi = \frac{H_i}{H_r}$$

The numerical values of τ_m are entered in Table (1).

Table (1). The mean skin-friction (τ_m)

k	$Gr = 2.0$	$Gr = 5.0$
1	1.65	5.81
2	3.95	10.78
3	5.00	15.68
4	7.11	20.58
5	9.89	25.58
6	11.21	30.58
7	13.89	35.58
8	15.46	40.58
9	17.46	45.58
10	19.46	50.58

It may be observed from this Table, the mean skin-friction increases with increase k and Gr both. It is interesting to note that the mean skin-friction is more affected by increase in Gr. This is interpreted physically as the presence of porous medium. The numerical values of the amplitude and phase of heat transfer are listed in Table (3).

Table 2. The values of (|M|) for k = 0.5

ω	$Gr = 2.0$, $\Pr = 7.0$	$Gr = 1.0$, $\Pr = 7.0$	$Gr = 2$, $\Pr = 0.71$	$Gr = 1.0$, $\Pr = 0.71$
2	45.53	23.27	0.929	0.912
4	57.65	28.85	0.622	0.754
6	64.78	31.45	0.521	0.605
8	72.24	35.46	0.437	0.452
10	96.65	49.02	0.422	0.225

Table 3. The amplitude and phase of heat transfer

ω	$\lvert H \rvert$		$\tan\psi$	
	$\Pr = 0.71$	$\Pr = 7.0$	$\Pr = 0.71$	$\Pr = 7.0$
2	1.254	3.964	0.447	1.016
4	1.574	5.592	0.673	1.095
6	1.957	6.574	1.153	1.055
8	2.528	7.985	1.432	1.000
10	3.268	8.564	1.567	1.000

We observe from the Table (3) that amplitude of heat transfer increases with increasing Pr and ω both. The values of amplitude are greater in case of water than in case of air. It is clear that there is always a phase lead for air (Pr = 0.71), whenever the phase of heat transfer almost remains same for the case of water (Pr = 7.0).

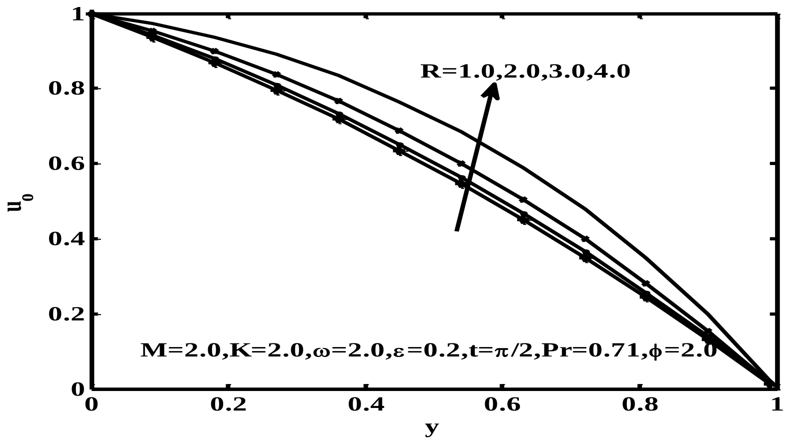

Figure (1). The mean flow velocity profiles

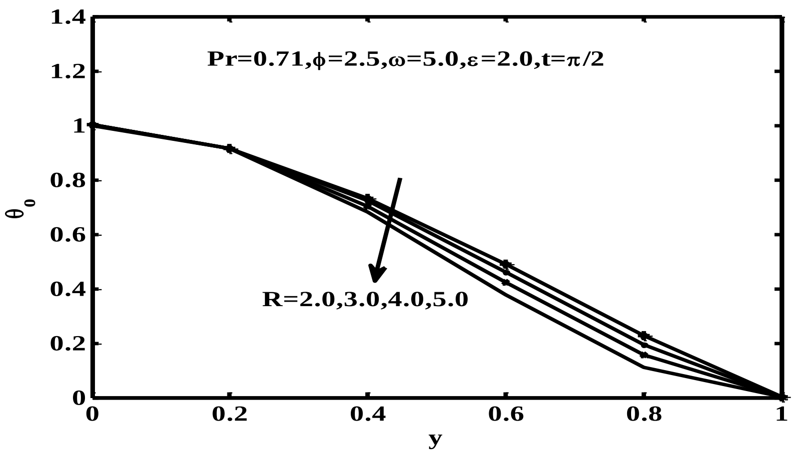

Figure (2). The mean flow temperature profiles

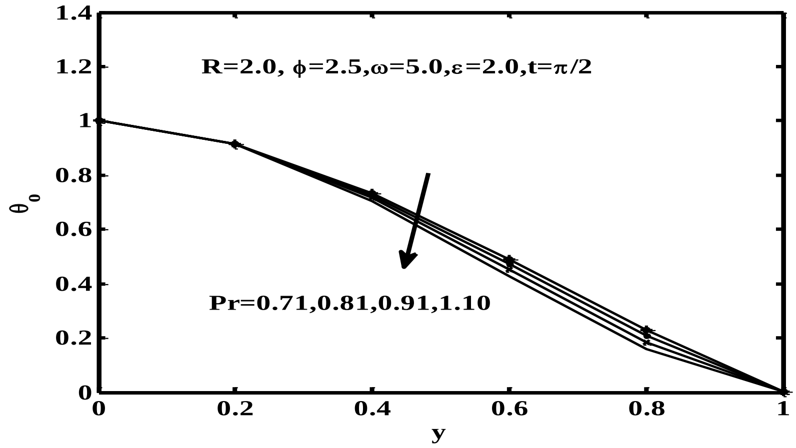

Figure (3). The mean flow temperature profiles

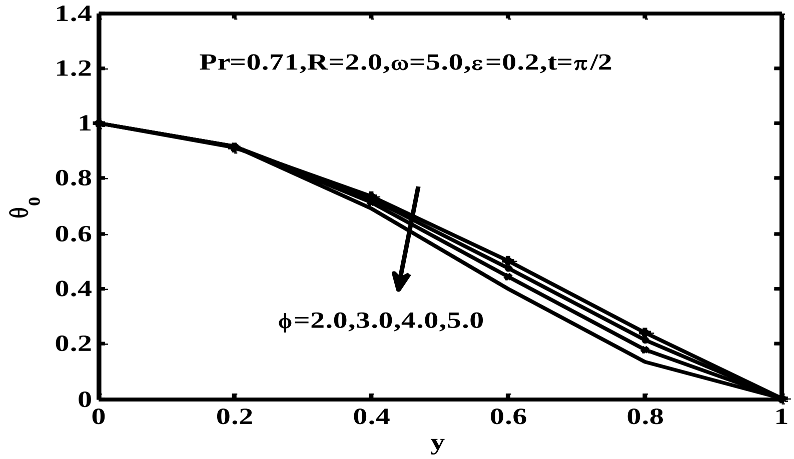

Figure (4). The mean temperature profiles

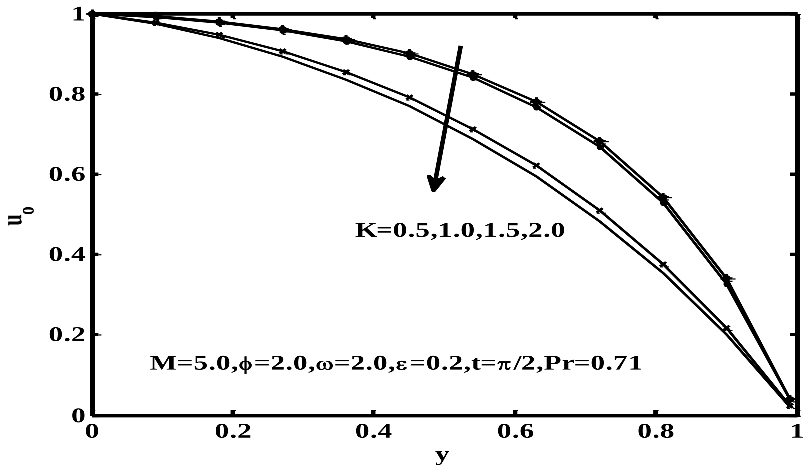

Figure (5). The mean flow velocity profiles

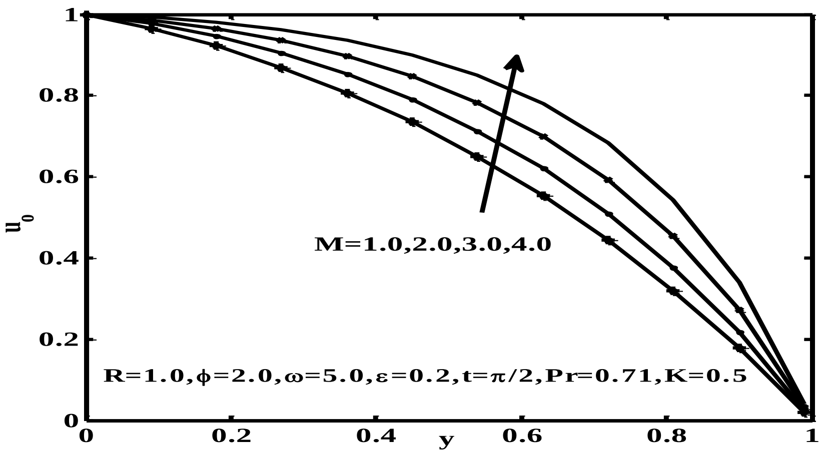

Figure (6). The mean flow velocity profiles

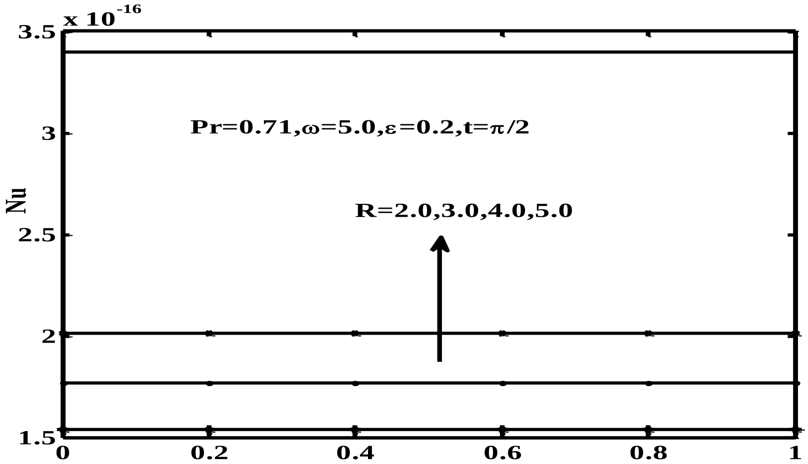

Figure. (7). The rate of heat transfer versus ϕ

Appendix

$$N_1 = \frac{1}{K} + M^2, N_2 = \Pr(\phi + R), N_3 = (N_3 + i\omega \Pr), \alpha = (N_1 + i\omega)$$

$$m_1 = \sqrt{N_1}, m_2 = -\sqrt{N_1},\ m_3 = \sqrt{\alpha}, m_4 = -\sqrt{\alpha},$$

$$m_5 = \sqrt{N_2}, m_6 = -\sqrt{N_2},\ m_7 = \sqrt{N_3}, m_8 = -\sqrt{N_3}$$

$$A_1 = \frac{1}{e^{m2} - e^{m1}}\left[2e^{m2} - J_1\left(e^{m2} - e^{m5}\right) - J_2\left(e^{m2} - e^{m6}\right) - 1\right]$$

$$A_2 = \frac{1}{e^{m1} - e^{m2}}\left[2e^{m1} - J_1\left(e^{m1} - e^{m5}\right) - J_2\left(e^{m1} - e^{m6}\right) - 1\right],$$

$$A_3 = \frac{N_1}{\alpha},\ A_4 = \frac{i\omega}{\alpha}$$

$$A_5 = -\frac{e^{m3} - A_3\left(e^{m3} - 1\right) - A_4\left(e^{m3} - 1\right)}{e^{m3} - e^{m4}},$$

$$A_6 = -\frac{e^{m4} - A_3\left(e^{m4} - 1\right) - A_4\left(e^{m4} - 1\right)}{e^{m4} - e^{m3}}$$

$$B_1 = \frac{e^{m5}}{e^{m5} - e^{m6}}, B_2 = \frac{e^{m6}}{e^{m6} - e^{m5}},\ B_3 = \frac{e^{m7}}{e^{m7} - e^{m8}}, B_4 = \frac{e^{m8}}{e^{m8} - e^{m7}}$$

$$J_1 = -\frac{GrB_2}{m_5^{\ 2} - N_1}, J_2 = -\frac{GrB_1}{m_6^{\ 2} - N_1}$$

Nomenclature

u^*	Velocity
U^*	Free-stream velocity
U_0	Mean constant free-stream velocity
ω^*	Frequency of oscillations
ρ	Density
μ	Viscosity
P	Pressure
g	Gravity
β	Volumetric coefficient of thermal expansion
k^*	Permeability parameter, thermal diffusivity
α	Temperature of fluid in the boundary layer
T_n^*	Temperature of the moving plate
T_s^*	Temperature of the stationary plate
B_0	Magnetic field strength
T^*	Time
B	Planck's function
q_r	Radiative heat flux
Gr	Grashof number
R	Radiation parameter
$\Pr$	Prandtl number
M	Magnetic parameter
ϕ	Heat source parameter
τ	Skin friction
τ_m	Mean Skin friction
N_u	Nusselt number

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