International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249–6890; ISSN (E): 2249–8001
Vol. 10, Issue 3, Jun2020, 5887–5900

Original Article

THE EFFECT OF RADIALLY VARYING MHD HEAT AND MASS TRANSFER ON PERISTALTIC FLOW OF WILLIAMSON FLUID IN A VERTICAL ANNULUS

K. VENUGOPAL REDDY[1], S. RAVIKUMAR[2], SK. ABZAL[3], M. RAJAIAH[4] & Y. HARNATH[5]

[1]Assistant Professor, Department of Mathematics, Vignan'sInstitute of Technology & Science, Deshmukhi village, Telangana, India

[2,3]Assistant Professor,Department of Mathematics, NBKR Institute of Science and Technology (Autonomous), Vidyanagar, SPSR Nellore, Andhra Pradesh, India

[4]Professor, Department of Mathematics, Audisankara College of Engineering &Technology(Autonomous), Gudur, SPSR Nellore, Andhra Prades, India

[5]Associate Professor, Department of Mathematics, Audisankara College of Engineering & Technology(Autonomous), Gudur, SPSR Nellore, Andhra Pradesh, India

ABSTRACT

In the present paper, we have investigated the influence of the effects of radially varying MHD and mass transfer on peristaltic flow of Williamson fluid model in a vertical annulus. The governing equations of Williamson fluid model are simplified using the assumptions of long wavelength and low Reynold's number. An approximated analytical solution has been derived for velocity field using Perturbation method. The expressions for pressure rise are calculated using numerical integration. The graphical results are presented to interpret various physical parameters.

KEYWORDS: Peristaltic Flow, Williamson Fluid, Annulus, Perturbation Solution, MHD

Received: Jun 10, 2020; **Accepted:** Jun 30, 2020; **Published:**Aug 01, 2020; **Paper Id.:** IJMPERDJUN2020560

I. INTRODUCTION

The study of peristaltic transport has enjoyed increased interest from investigators in several engineering disciplines. From a mechanical point of view, peristalsis offers the opportunity of constructing pumps in which the transported medium does not come in direct contact with any moving parts such as valves, plungers, and rotors. This could be of great benefit in cases where the medium is either highly abrasive or decomposable under stress. This has led to the development of fingers and roller pumps which work according to the principle of peristalsis. Applications include dialysis machines, open-heart bypass pump machines, and infusion pumps. After the first investigation reported by Latham [1], several theoretical and experimental investigations [2–5] about the peristaltic flow of Newtonian and non-Newtonian fluids have been made under different conditions with reference to physiological and mechanical situations. The peristaltic transport of magnetohydrodynamic (MHD) flow of a fluid in a channel is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g., the blood, blood pump machines and with the need for experimental as well as theoretical research on the operation of a peristaltic MHD compressor. Effect of a moving magnetic field on blood flow was investigated by Stud et al. [6], and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Agarwal and Anwaruddin[7] developed a mathematical model of MHD flow of blood through an equally branched channel with flexible walls executing peristaltic waves using long wave length approximation method and observed, for the flow blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations. The principle of magnetic field is successfully applied to Magnetic Resonance Imaging (MRI) when a patient under goes in a height static magnetic field. Abbasi et al. [8] developed a mathematical model on peristaltic transport of MHD fluid by considering variable viscosity. Moreover, the influence of magnetic field on peristaltic flow of a Casson fluid in an asymmetric channel was studied by Akbar[9]. Mahmoud et al. [10] have also examined Effect of porous media and magnetic field on peristaltic transport of a Jeffrey fluid in an asymmetric channel.

The peristalsis in the presence of heat transfer is imperative in many processes as oxygenation and hemodialysis. Heat transfer is also significant in the treatment of diseased tissues in cancer. Furthermore, the human lungs, bile duct and gall bladder have stones that behave like natural porous media. Also, keeping in mind the pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the coronary artery may be considered as the domains of porous medium. The magnetohydrodynamic peristaltic flow in a channel has a pivotal role in the motion of physiological fluids including blood and blood pump machines. Mass transfer in peristaltic flow occurs during the chemical breakdown of food, amalgamation of gastric juices with food and in other digestion processes. Motivated by these facts, Akbar[11] has carried out the influence of magnetic field on flow and heat transfer of a carbon nanotube induced by peristaltic waves and observed that with the increase of solid volume fraction of the nanoparticles and heat absorption parameter, the temperature profile increases significantly and also different authors have investigated the influence of heat transfer and magnetic field on the peristaltic transport of Newtonian fluid and Non-Newtonian fluids with different geometries from[12-17].Recently, few attempts have been made in the peristaltic literature to study the combined effects of heat and mass transfer. Eldabe et al. [18] analyzed the mixed convective heat and mass transfer in a non-Newtonian fluid at a peristaltic surface with temperature-dependent viscosity. Ogulu [19] examined heat and mass transfer of blood under the influence of a uniform magnetic field. Very recently, Nadeem and Noreen Akbar[20] have investigated the effects of heat and mass transfer peristaltic flow of Williamson model in a vertical annulus. The main aim of the present study is to provide an analytical solution for the peristaltic flow of a non-Newtonian fluid under long wavelength and low Reynold's number considerations. The governing equations are constituted for Williamson fluid with radially varying. Exact solutions have been calculated for energy and equation of concentration and also analytical solution has been presented for velocity profile. At the end of the article graphical results have been presented for various parameter of interest.

2.MATHEMATICAL FORMULATION

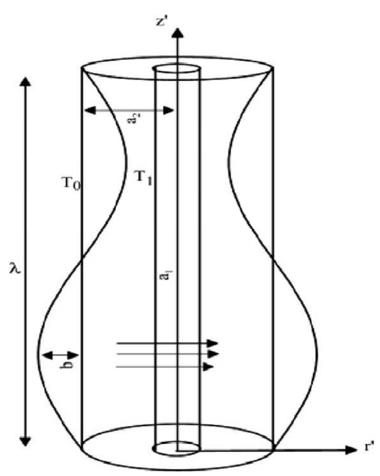

Figure 1: Geometry of the problem

For an incompressible fluid the balance of mass and momentum are given by

$$div V = 0 \tag{1}$$

$$\rho \frac{dV}{dt} = div\, S + \rho f \tag{2}$$

where ρ is the density, V is the velocity vector, S is theCauchy stress tensor, f represents the specific body force and $\frac{d}{dt}$ represents the material time derivative. The constitutive equation is given by

$$S = -PI + \tau \tag{3}$$

$$\tau = -\left[\eta_\infty + (\eta_o + \eta_\infty)(1 - \Gamma\bar{\dot{\gamma}})^{-1}\right]\bar{\dot{\gamma}} \tag{4}$$

in which $-PI$ is the spherical part of the stress due to constraint of incompressibility, τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero rate viscosity, Γ is the time constant,and $\bar{\dot{\gamma}}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{5}$$

$$\tau = -\eta_o\left[(1 - \Gamma\bar{\dot{\gamma}})^{-1}\right]\bar{\dot{\gamma}} = -\eta_o\left[(1 - \Gamma\bar{\dot{\gamma}})\right]\bar{\dot{\gamma}} \tag{6}$$

The peristaltic transport of an incompressible Williamson fluid with radially varying MHD in a vertical annulus is considered. The inner tube is rigid and maintained at temperature $\bar{T}_1$, the outer tube has a sinusoidal wave travelling down its walls and maintained at temperature $\bar{T}_o$.

The geometry of the wall surface is defined as

$$\bar{R}_1 = a_1 \tag{7}$$

$$\bar{R}_2 = a_2 + b\sin\frac{2\pi}{\lambda}\left(\bar{Z} - c\bar{t}\right) \tag{8}$$

where a_1 is the radius of the inner tube, a_2 is the radius of the outer tube at inlet, b is the wave amplitude, λ is the wavelength, c the wave speed and $\bar{t}$ the time.

The governing equations in the fixed frame for an incompressible Williamson fluid model are given as

$$\frac{\partial \bar{U}}{\partial \bar{R}} + \frac{\bar{U}}{\bar{R}} + \frac{\partial \bar{W}}{\partial \bar{Z}} = 0\ (9)\ \rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{R}} + \bar{W}\frac{\partial}{\partial \bar{Z}}\right)\bar{U} = -\frac{\partial \bar{P}}{\partial \bar{R}} - \frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}\left(\bar{R}\bar{S}_{\bar{R}\bar{R}}\right) - \frac{\partial}{\partial \bar{Z}}\left(\bar{S}_{\bar{R}\bar{Z}}\right) - \frac{\bar{S}_{\bar{\theta}\bar{\theta}}}{\bar{R}} \tag{10}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}} + \bar{U}\frac{\partial}{\partial \bar{R}} + \bar{W}\frac{\partial}{\partial \bar{Z}}\right)\bar{W} = -\frac{\partial \bar{P}}{\partial \bar{Z}} - \frac{1}{\bar{R}}\frac{\partial}{\partial \bar{R}}\left(\bar{R}\bar{S}_{\bar{R}\bar{Z}}\right) - \frac{\partial}{\partial \bar{Z}}\left(\bar{S}_{\bar{Z}\bar{Z}}\right) + \rho g\alpha\left(\bar{T} - \bar{T}_o\right) + \rho g\alpha\left(\bar{C} - \bar{C}_o\right) - \bar{B}_o^2\bar{r}\left(J + c\right) \tag{11}$$

The energy equation in absence of dissipation terms and the concentration equation are defined as

$$\rho c_p\left(\frac{\partial}{\partial \bar{t}}+\bar{U}\frac{\partial}{\partial \bar{R}}+\bar{W}\frac{\partial}{\partial \bar{Z}}\right)\bar{T}=k\left(\frac{\partial^2 \bar{T}}{\partial \bar{R}^2}+\frac{1}{\bar{R}}\frac{\partial \bar{T}}{\partial \bar{R}}+\frac{\partial^2 \bar{T}}{\partial \bar{Z}^2}\right) \tag{12}$$

$$\left(\frac{\partial}{\partial \bar{t}}+\bar{U}\frac{\partial}{\partial \bar{R}}+\bar{W}\frac{\partial}{\partial \bar{Z}}\right)\bar{C}=D\left(\frac{\partial^2 \bar{C}}{\partial \bar{R}^2}+\frac{1}{\bar{R}}\frac{\partial \bar{C}}{\partial \bar{R}}+\frac{\partial^2 \bar{C}}{\partial \bar{Z}^2}\right)+\frac{DK_T}{T_m}\left(\frac{\partial^2 \bar{T}}{\partial \bar{R}^2}+\frac{1}{\bar{R}}\frac{\partial \bar{T}}{\partial \bar{R}}+\frac{\partial^2 \bar{T}}{\partial \bar{Z}^2}\right) \tag{13}$$

In In the above equations, $\bar{P}$ is the pressure, $\bar{U},\bar{W}$ are the respective velocity components in the radial and axialdirections respectively, $\bar{T}$ is the temperature, $\bar{C}$ is the concentration of fluid, ρ is the density, k denotes the thermal conductivity, c_p is the specific heat at constantpressure T_m is the temperature of the medium, D is the coefficient of mass diffusivity, K_T is the thermal diffusion ratio.

In the fixed coordinates $(\bar{R},\bar{Z})$, the flow between the two tubes is unsteady.It becomes steady in a wave frame $(\bar{r},\bar{z})$ moving with the same speed as the wave moves in the $\bar{Z}-$direction. The transformations between the two frames are

$$\begin{aligned}&\bar{r}=\bar{R},\quad \bar{z}=\bar{Z}-c\bar{t},\\&\bar{u}=\bar{U},\quad \bar{w}=\bar{W}-c,\end{aligned} \tag{14}$$

where $\bar{u}$ and $\bar{w}$ are the velocities in the wave frame.

The appropriate boundary conditions in the wave frame are of the following form

$$\left.\begin{aligned}&\bar{w}=-1 \quad at\ \bar{r}=\bar{r}_1\\&\bar{w}=-1 \quad at\ \bar{r}=\bar{r}_2\\&\bar{T}=\bar{T}_o \quad at\ \bar{r}=\bar{r}_1\\&\bar{T}=\bar{T}_1 \quad at\ \bar{r}=\bar{r}_2\\&\bar{C}=\bar{C}_0 \quad at\ \bar{r}=\bar{r}_1\\&\bar{C}=\bar{C}_1 \quad at\ \bar{r}=\bar{r}_2\end{aligned}\right\} \tag{15}$$

Introduce the non-dimensional variables:

$$\left.\begin{aligned}
&R=\frac{\bar{R}}{a_2},\quad r=\frac{\bar{r}}{a_2},\ Z=\frac{\bar{Z}}{\lambda},\quad z=\frac{\bar{z}}{\lambda},W=\frac{\bar{W}}{c},\quad w=\frac{\bar{w}}{c},\dot{\gamma}=\frac{a_2}{c}\bar{\dot{\gamma}},U=\frac{\lambda\bar{U}}{a_2 c},\\
&u=\frac{\lambda\bar{u}}{a_2 c},P=\frac{a_2^2\bar{P}}{c\lambda\mu},\theta=\frac{\left(\bar{T}-\bar{T}_1\right)}{\left(\bar{T}_o-\bar{T}_1\right)},t=\frac{c\bar{t}}{\lambda},\delta=\frac{a_2}{\lambda},\mathrm{Re}=\frac{\rho c a_2}{\mu},S=\frac{a_2\bar{S}}{c\mu},\\
&r_1=\frac{\bar{r}_1}{a_2}=\varepsilon,r_2=\frac{\bar{r}_2}{a_2}=1+\varphi\sin\left(2\pi z\right),\beta=\frac{Q_o a_2^2}{k\left(\bar{T}_1-\bar{T}_o\right)},Gr=\frac{g\alpha a_2^3\left(\bar{T}_1-\bar{T}_o\right)}{\upsilon^2},\\
&Br=\frac{\alpha g a^3\left(\bar{C}_1-\bar{C}_o\right)}{\upsilon^2},Sr=\frac{\rho D K_T\left(\bar{T}_o-\bar{T}_1\right)}{\mu T_m\left(\bar{C}_o-\bar{C}_1\right)},Sc=\frac{\mu}{D\rho},\sigma=\frac{\left(\bar{C}-\bar{C}_1\right)}{\left(\bar{C}_o-\bar{C}_1\right)},\\
&\varphi=\frac{b}{a_2},M\left(r\right)=B_o\left(r\right)\sqrt{\frac{\sigma}{\mu}}a_2
\end{aligned}\right\}\qquad(16)$$

Making use of (14) and (16),(9) to (13) take the form

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0\ (17)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u=-\frac{\partial P}{\partial r}-\frac{\delta}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)-\delta^2\frac{\partial}{\partial z}\left(\tau_{rz}\right)\ (18)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)w=-\frac{\partial P}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)-\delta\frac{\partial}{\partial z}\left(\tau_{zz}\right)+G_r\theta+B_r\sigma-\left(M\left(r\right)\right)^2\left(w+1\right)\qquad(19)$$

$$\mathrm{Re}\,\delta\,\mathrm{Pr}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\theta=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\delta^2\frac{\partial^2\theta}{\partial z^2}+\beta\qquad(20)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\sigma=\frac{1}{S_c}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sigma}{\partial r}\right)+\delta^2\frac{\partial^2\sigma}{\partial z^2}\right)+S_r\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\delta^2\frac{\partial^2\theta}{\partial z^2}\right)\qquad(21)$$

where

$$\tau_{rr}=-2\delta[1+We\,\dot{\gamma}]\frac{\partial u}{\partial r}\quad\tau_{rz}=-[1+We\dot{\gamma}]\left(\frac{\partial u}{\partial r}\delta^2+\frac{\partial w}{\partial r}\right)\tau_{zz}=-2\delta[1+We\,\dot{\gamma}]\delta\frac{\partial w}{\partial z}$$

$$\dot{\gamma}=\left[2\delta^2\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial u}{\partial z}\delta^2+\frac{\partial w}{\partial r}\right)^2+2\delta^2\left(\frac{\partial w}{\partial z}\right)^2\right]^{\frac{1}{2}}$$

in which δ,Re,We represent the wave, Reynolds and Weissenberg numbers, respectively. Under the assumptions of long wavelength $\delta<<1$ and low Reynolds number, neglectingthe terms of order δ and higher,(18)-(21) and boundary condition (15) take the form

$$\frac{\partial P}{\partial r}=0 \tag{22}$$

$$\frac{\partial P}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1+We\frac{\partial w}{\partial r}\right)\frac{\partial w}{\partial r}\right]+G_r\theta+B_r\sigma-\left(M\left(r\right)\right)^2\left(w+1\right) \tag{23}$$

$$0=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\beta \tag{24}$$

$$0=\frac{1}{Sc}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sigma}{\partial r}\right)\right)+Sc\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)\right) \tag{25}$$

$$\left.\begin{array}{l} w=-1\,at\,r=r_1=\varepsilon \\ w=-1\,at\,r=r_2=1+\varphi\sin\left(2\pi z\right) \\ \theta=1\;at\,r=r_1,\theta=0\,at\,r=r_2 \\ \sigma=1\,at\,r=r_1,\sigma=0\,at\;r=r_2 \end{array}\right\} \tag{26}$$

In the forthcoming analysis variable set $M\left(r\right)=\frac{M}{r}$

where is the M is the Hartmann number, Sr is the Soret number, Sc Schmidt number, Br is the local concentration Grashof number, μ is the viscosity of the fluid, r_2 dimensionless form of radius of outer tube, ϕ amplitude ratio and Gr is the local temperature Grashof number.

3.PERTURBATION SOLUTION

Solving (24) and (25) subject to the boundary conditions (26), we obtain the expression for temperature and concentration field as follows

$$\theta(r,z)=\frac{1}{a_{11}}\left(a_{12}\ln(r)+a_{13}r^2+a_{14}\right) \tag{27}$$

$$\sigma\left(r,z\right)=-\frac{S_rS_c}{a_{11}}\left(a_{12}\ln\left(r\right)+a_{13}r^2+a_{14}\right)+a_{17}\ln r+a_{18} \tag{28}$$

To get the solution of (23) we employ the regular perturbation to find the solution. For perturbation solution, we expand w, F and P as

$$w=w_0+We\,w_1+O\left(We^2\right) \tag{29}$$

$$F=F_0+We\,F_1+O\left(We^2\right) \tag{30}$$

$$P=P_0+We\,P_1+O\left(We^2\right) \tag{31}$$

The perturbation results for small parameter We, satisfying the conditions (26) for velocity and pressure gradient can be written as

$$w(r,z) = c_1 r^M + c_2 r^{-M} + \frac{dp_o}{dz} a_{39} r^2 + a_{35} r^2 \ln(r) + a_{36} r^2 + a_{37} r^4 - 1 + c_3 r^M + c_4 r^{-M}$$

$$+ \frac{dp_1}{dz} a_{39} r^2 - (y_{p_{13}})_r - (y_{p_{14}})_r$$

$$\frac{dp}{dz} = r_2^2 \left(\frac{-1}{L_2}\right) F + L_{13}$$

where

$$a_{11} = 4(\ln(r_1) - \ln(r_2)), a_{12} = 4 + \beta(r_1^2 - r_2^2), a_{13} = \beta(\ln(r_2) - \ln(r_1)),$$

$$a_{14} = \beta(r_2^2 \ln(r_1) - r_1^2 \ln(r_2)) - 4\ln(r_2), a_{15} = -\frac{SrSc}{a_{11}}(a_{12}\ln(r_1) + a_{13} r_1^2 + a_{14}),$$

$$a_{16} = -\frac{SrSc}{a_{11}}(a_{12}\ln(r_2) + a_{13} r_2^2 + a_{14}), \ a_{17} = \frac{1 - a_{15} - a_{16}}{\ln(r_1) - \ln(r_2)}, \ a_{18} = 1 - a_{15} - a_{17}\ln(r_1) \ a_{19} = \frac{(Gra_{13})}{a_{11}(16 - M^2)},$$

$$a_{20} = \frac{(Gra_{14})}{a_{11}(4 - M^2)}, \ a_{21} = \frac{-4a_{18}}{4 - M^2}, a_{22} = a_{21} + a_{20},$$

$$a_{23} = \frac{-BrSrSc}{a_{11}} \ a_{24} = \frac{a_{12}}{(4 - M^2)} \ a_{25} = \frac{-4a_{12}}{(4 - M^2)^2} \ a_{26} = \frac{a_{13}}{16 - M^2}$$

$$, a_{27} = \frac{a_{14}}{4 - M^2}, \ a_{28} = \frac{Bra_{17}}{4 - M^2}, a_{29} = \frac{-4\,Bra_{17}}{(4 - M^2)^2}, a_{30} = \frac{Bra_{18}}{4 - M^2},$$

$$a_{31} = a_{23} + a_{24}, a_{32} = a_{31} + a_{28}, a_{33} = a_{23}(a_{25} + a_{27}) + (a_{29} + a_{30}), a_{34} = a_{23} a_{26},$$

$$a_{35} = -(a_{18} + a_{32}), a_{36} = -(a_{19} + a_{34}), a_{37} = -(a_{19} + a_{34}), a_{38} = a_{35} + 2a_{36}, a_{39} = \frac{1}{4 - M^2},$$

$$a_{40} = M(M-1)c_1, a_{41} = M(M+1)c_2, \ a_{42} = \frac{a_{12}}{a_{11}}, a_{43} = \frac{5a_{13}}{a_{11}}, \ a_{44} = 3a_{14} + a_{12},$$

$$a_{45} = a_{41} + 2a_{12}, a_{46} = a_{39} a_{12} a_{63}, a_{47} = a_{40} a_{12} a_{63}, a_{48} = 2a_{38} a_{12} a_{63}, a_{49} = a_{35} a_{12} a_{63}$$

$$a_{50} = 2a_{36} a_{12} a_{63}, a_{51} = 12a_{37} a_{12} a_{63}, a_{52} = a_{13} a_{39} a_{63}, a_{53} = a_{40} a_{13} a_{63}, a_{54} = 2a_{13} a_{38} a_{63},$$

$$a_{55} = a_{13} a_{35} a_{63}, a_{56} = 2a_{36} a_{13} a_{63}, a_{57} = 12a_{37} a_{13} a_{63}, a_{58} = a_{14} a_{39} a_{63}, a_{59} = a_{14} a_{40} a_{63},$$

$$a_{60}=a_{14}a_{38}, a_{61}=a_{14}a_{35}, a_{62}=2a_{36}a_{14},\ a_{63}=\frac{1}{a_{11}}, a_{64}=12a_{37}a_{14}$$

$$w_o=c_1r^M+c_2r^{-M}+\frac{dp_o}{dz}a_{39}r^2+a_{35}r^2\log(r)+a_{36}r^2+a_{37}r^4-1 \quad (32)$$

$$c_{21}=r_2^{-M}r_1^M-r_1^{-M}r_2^M, c_{22}=\left(r_1^2r_2^M-r_2^2r_1^M\right)a_{39},$$

$$c_{23}=a_{35}\left(r_1^2r_2^M\log(r_1)-r_2^2r_1^M\log(r_2)\right)+a_{36}\left(r_1^2r_2^M-r_2^2r_1^M\right)+a_{37}\left(r_1^4r_2^M-r_2^4r_1^M\right)$$

$$c_1=-r_1^{(-M)}\left(c_2r_1^{-M}+\frac{dp_o}{dz}a_{39}r_1^2+a_{35}r_1^2\log(r_1)\right)+a_{36}r_1^2+a_{37}r_1^4, c_2=\frac{1}{c_{21}}\left(\frac{dp_o}{dz}c_{22}+c_{23}\right)$$

$$c_{41}=r_2^{-M}r_1^M-r_1^{-M}r_2^M \quad , \quad c_{42}=\left(r_1^2r_2^M-r_2^2r_1^M\right) \quad , \quad c_4=\frac{1}{c_{41}}\begin{pmatrix} c_{42}\frac{dp_1}{dz}a_{39}-r_2^My_{p_{13r_1}}-r_1^My_{p_{13r_2}} \\ -r_2^My_{p_{14r_1}}-r_1^My_{p_{14r_2}}\end{pmatrix}$$

$$c_3=r_1^{-M}\left(c_4r_1^{-M}\frac{dp_1}{dz}a_{39}r_1^2-y_{p_{13r_1}}-y_{p_{14r_2}}\right)\ y_{p_{13r}}=r^2\frac{\partial}{\partial r}\left(\frac{\partial w_o}{\partial r}\right)^2,\quad y_{p_{14r}}=r\left(\frac{\partial w_o}{\partial r}\right)^2,$$

$$w_1=c_3r^M+c_4r^{-M}+\frac{dp_1}{dz}a_{39}r^2-y_{p_{13r}}-y_{p_{14r}} \quad (33)$$

$$L_1=\frac{c_1}{2+M}\left(r_2^{2+M}-r_1^{2+M}\right)+\frac{c_2}{2-M}\left(r_2^{2-M}-r_1^{2-M}\right), L_2=\frac{a_{38}}{4}\left(r_2^4-r_1^4\right),$$

$$L_3=\frac{a_{35}}{16}\left(r_2^4\left(4\log(r_2)-1\right)-r_1^4\left(4\log(r_1)-1\right)\right)+\frac{a_{36}}{20}\left(r_2^5-r_1^5\right)+\frac{a_{37}}{42}\left(r_2^7-r_1^7\right)-\frac{1}{6}\left(r_2^3-r_1^3\right)$$

$$L_4=-(L_1+L_3)$$

4.RESULTS AND DISCUSSIONS

Fig.2. shows that the velocity field using perturbation solution. In this section we have presented the solution of the Williamson fluid model graphically. The expression for pressure rise ΔP is calculated numerically using Mathematica software. The effects of various parameters on the pressure rise ΔP are shown in figures (3-7) for different values of Weissenberg number (We), amplitude ratio (ϕ),Thermal ratio (β),Radius value $(\varepsilon)$and Hartmann number (M). It is observed that the pressure rise ΔP decreases with the increase in $We, \phi, \beta, \varepsilon$ and M .It is noticed that best peristaltic pumping region with the Hartmann number M from Fig.7. Otherwise there is augmented pumping.

The frictional force on inner tube $\left(F^{(o)}\right)$ are observed from the Fig.8. to Fig.12. for various parameter values of Weissenberg number (We), amplitude ratio (ϕ),Thermal ratio (β),Radius value $(\varepsilon)$and Hartmann number (M). As the values of We, ε and β are increasing. We observed that the plots for $F^{(o)}$ is decreasing from Fig. 8, Fig. 10 and Fig. 12.

while the figures Fig.9. andFig.13. are depicted that increasing as the values of ϕ and M are increasing. The frictional force on outer tube $\left(F^{(i)}\right)$ are observed from the Fig.13. to Fig.17. for various parameter values of Weissenberg number (We), amplitude ratio (ϕ),Thermal ratio (β),Radius value $(\varepsilon)$and Hartmann number(M).It is found that, the $F^{(i)}$ is decreasing as the various values of We and β from Fig.13.and Fig.16.while it is increasing for the different values of ϕ, ε and M .

The Temperature of the field is increasing with the different values of β whereas the concentration of the field decreasing for the various values of β, Sr and Sc . The fixed constant values for Pressure Rise ΔP , frictional force on inner tube $F^{(o)}$ and frictional force on outer tube $F^{(i)}$ are given by $Gr = 2,\ We = 0.1,\ M = 0.3,\ Sr = 5,\ Sc = 0.2,\ Sc = 0.2,$ $\varepsilon = 0.1,\ z = 0.1,\ \frac{dp}{dz} = 0.3,\ \beta = 0.04$

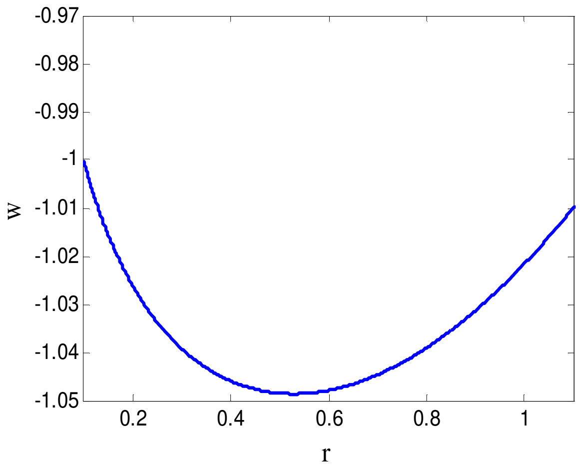

Figure 2: Velocityfor $G_r = 0.01, We = 0.001,$ **,** $Sr = 0.5,$ $Sc = 0.3$ **,** $Br = 0.01$ **,** $\varepsilon = 0.1$ **,** $z = 0.1,\ \frac{dp_o}{dz} = 0.3,\ \frac{dp_1}{dz} = 0.3,$ $\frac{dp}{dz} = 0.3$ **,** $\varphi = 0.3,\ \beta = 0.08$

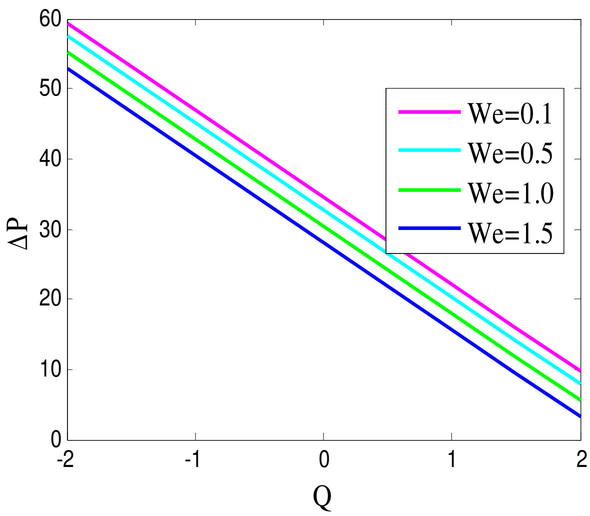

Figure 3: Effect of We **on Pressure rise**

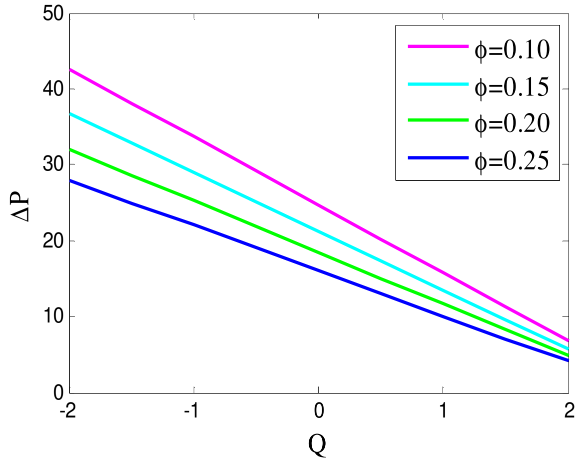

Figure 4: Effect of φ on Pressure rise

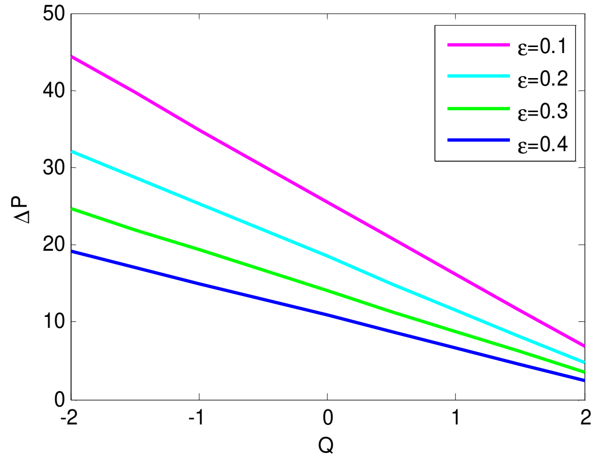

Figure 5: Effect of ε on Pressure rise

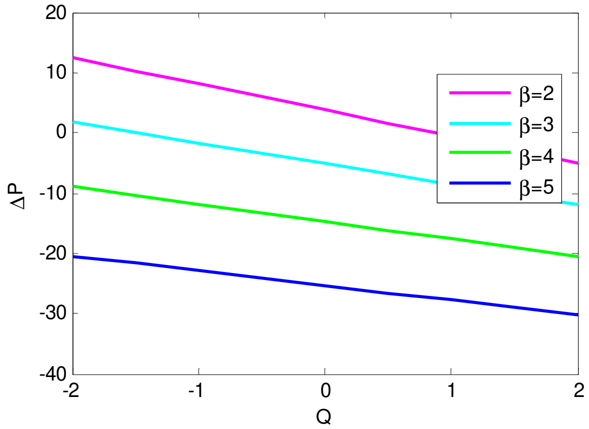

Figure 6: Effect of β on Pressure rise

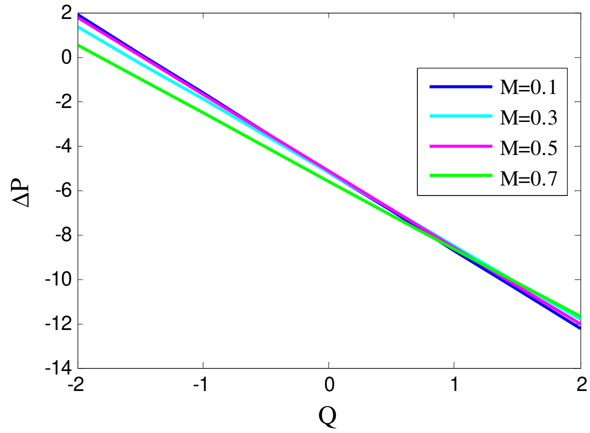

Figure 7: Effect of M on Pressure rise

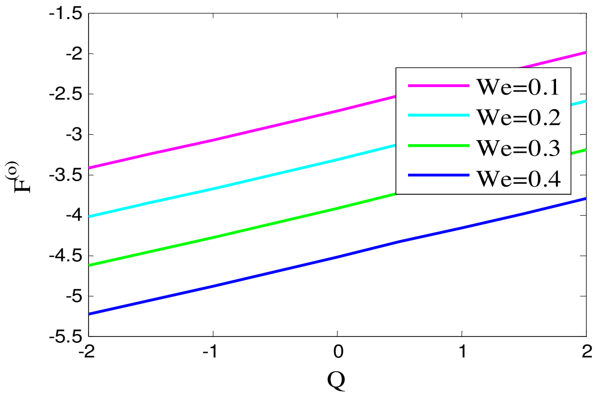

Figure 8: Effect of We on Frictional force(inner tube)

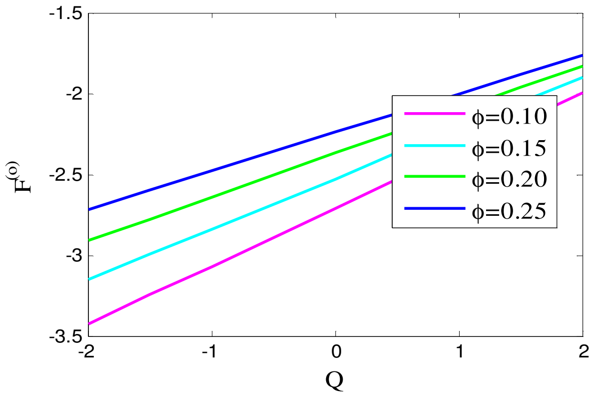

Figure 9: Effect of ϕ on Frictional force(inner tube)

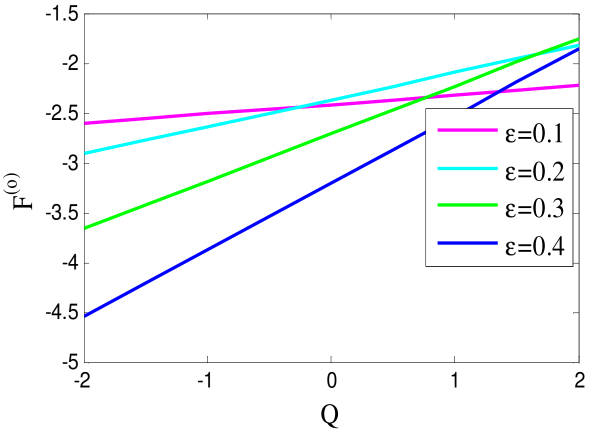

Figure 10: Effect of ε on Frictional force (inner tube)

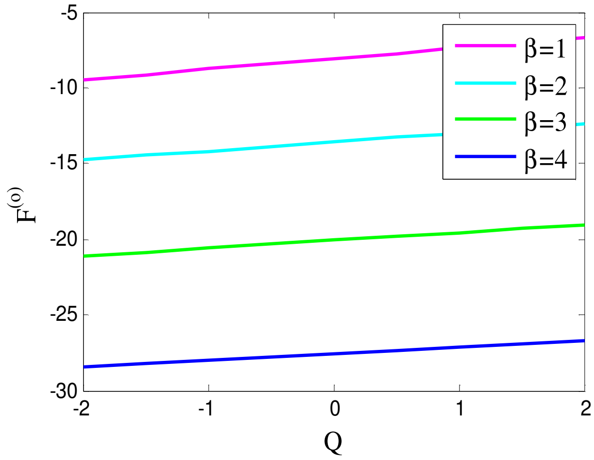

Figure 11: Effect of β on Frictional force (inner tube)

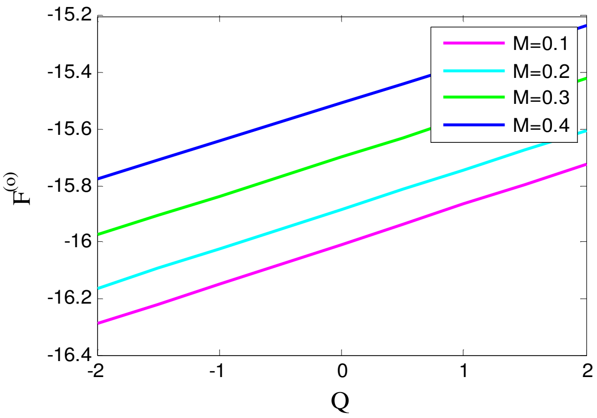

Figure 12: Effect of M on Frictional force (inner tube)

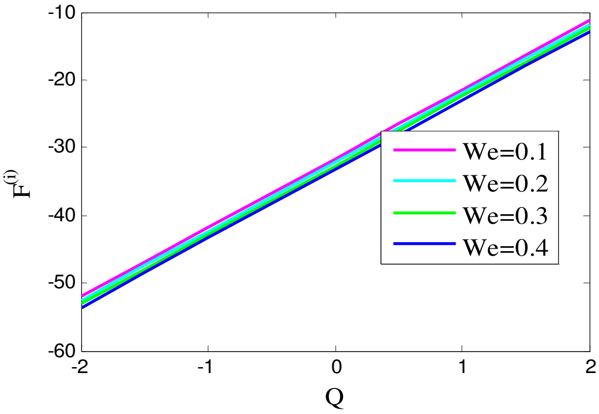

Figure 13: Effect of We on Frictional force (outer tube)

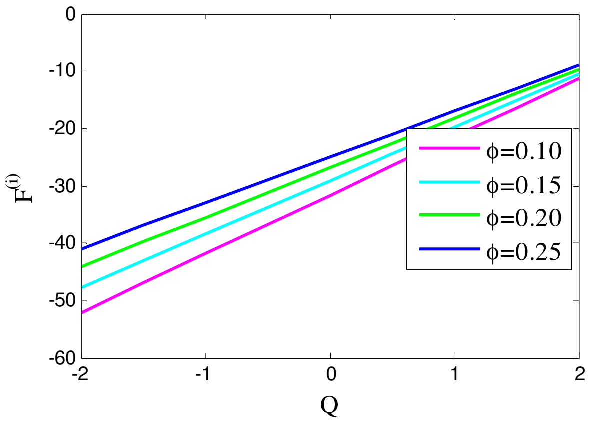

Figure 14: Effect of ϕ on Frictional force (outer tube)

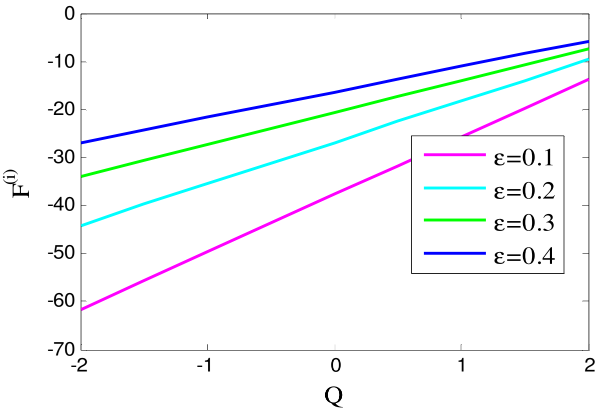

Figure 15: Effect of ε on Frictional force (outer tube)

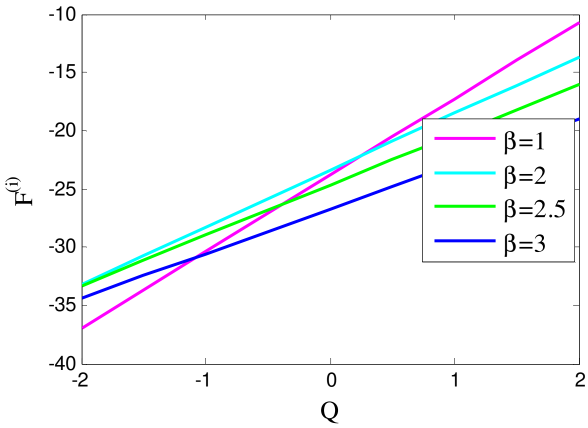

Figure 16: Effect of β on Frictional force (outer tube)

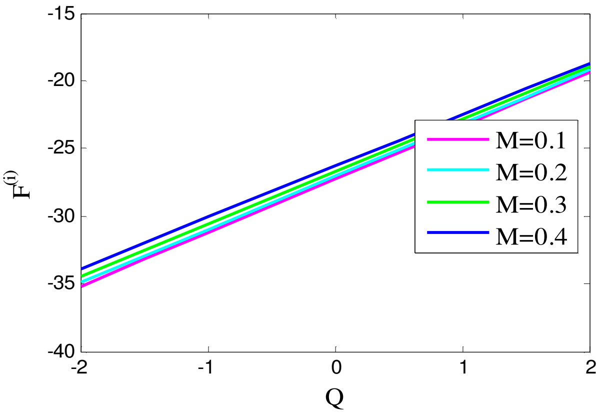

Figure 17:Effect of M on Frictional force (outer tube)

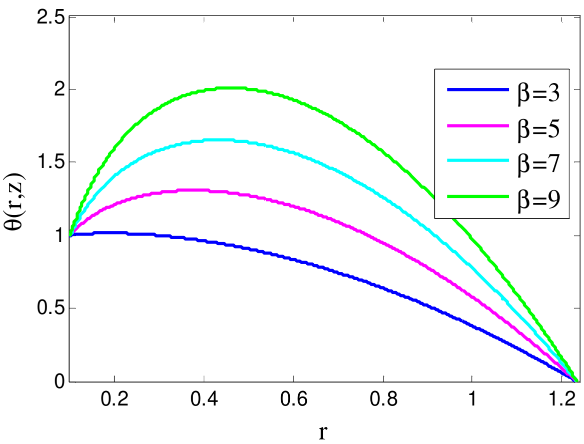

Figure 18: Temperature field for different values of β.

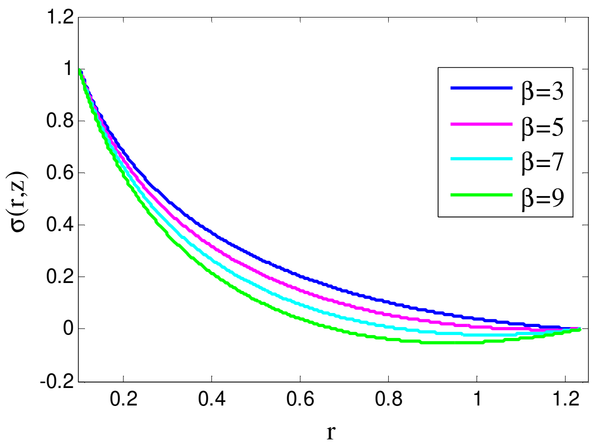

Figure 19: Concentration for different values of β.

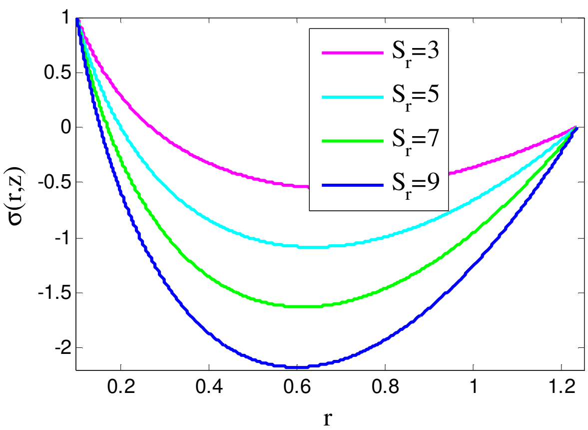

Figure 20: Concentration for different values of Sr.

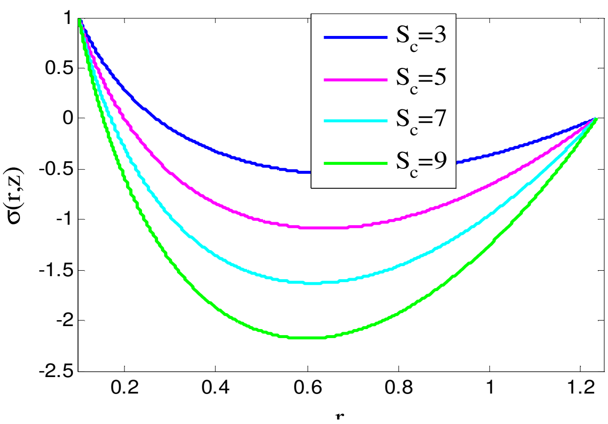

Figure 21: Concentration for different values of Sc.

5. CONCLUSIONS

In this paper, we have investigated the influence of the effect of radially varying MHD andmass transfer on peristaltic flow of Williamson fluidin a vertical annulus under the assumptions of low Reynolds number and long wave length approximation. The analytical solutions are obtained for the angular velocity using Perturbation method and solution for

Pressure Rise is calculated using Numerical integration. The behaviors of the flow characteristics are analyzed through graphs.

- The angular velocity of the fluid is curved channel in the annulus.
- The magnitude of the pressure rise ΔP decreases with the increase in $We, \phi, \beta, \varepsilon$ and M .
- The frictional force on inner tube $\left(F^{(o)}\right)$ is increasing as the values of ϕ and M areincreasing.
- The frictional force on outer tube $\left(F^{(i)}\right)$ is decreasing as the various values of We and β.
- If $M = 0$,it is good agreement with Reference[25].

REFERENCES

1. *Latham, T. W. (1966). Fluid Motion in a Peristaltic Pump. MIT Press, Cambridge, Mass, USA.*
2. *Shapiro, A. H., Jaffrin, M. Y., &Weinberg, S. L. (1969). Peristaltic pumping with long wave lengths at low Reynolds number. Journal of Fluid Mechanics, 37, 799 - 825.*
3. *Senapati, Nityananda, and Rajendra Kumar Dhal. "Effect of slip condition on unsteady MHD oscillatory flow in a channel filled with porous medium with heat radiation and mass transfer." International Journal of Applied Mathematics and Statistical Sciences 2 (2013): 11-20.*
4. *Radhakrishnamacharya, G &Radhakrishnamurty, V. (1993). Heat transfer to peristaltic transport in a non-uniform channel. Defence Science Journal, 43, 275 - 280.*
5. *Hayat, T., Wang, Y., Hutter,K.,Asghar, S., &Siddiqui, A. M. (2004). Peristaltic transport of an Oldroyd-B fluid in a planar channel.Mathematical Problems in Engineering, 2004, 347 - 376.*
6. *Poonia, Hemant, and R. C. Chaudhary."Mass transfer with chemical reaction effects on MHD free convective flow past an accelerated vertical plate embedded in a porous medium." Int. J. of Applied Mathematics & Statistical Sciences 5 (2016).*
7. *Vajravelu, K., Sreenadh, S., &Babu, V. R. (2005).Peristaltic transport of a Herschel-Bulkley fluid in an inclined tube.International Journal of Non-Linear Mechanics, 40, 83 - 90.*
8. *Sud, V. K., Sekhon, G. S., & Mishra, R. K. (1977). Pumping action on blood flow by a magnetic field.Bull. Math. Biol., 39, 385 - 390.*
9. *Agarwal, H. L., & B. Anwaruddin, "Peristaltic flow of blood in a branch", Ranchi Univ.Math. J.15 (1984) 111–121.*
10. *Senapati, N., R. K. Dhal, and K. Ray. "Effect of chemical reaction on MHD free convection on Kuvshinshki fluid through porous medium in presence of heat radiation with constant heat and mass flux across moving plate." International Journal of Applied Mathematics & Statistical Sciences 2.4 (2013): 45-52.*
11. *Abbasi, F. M., Hayat,T., Alsaedi, A., &Ahmed, B. (2014). Soret and Dufour effects on peristaltic transport of MHD fluid with variable viscosity. Appl. Math. Inf. Sci.,8, 211 - 219.*
12. *Akbar, N. S. (2015). Influence of magnetic field on peristaltic flow of a Casson fluid in an asymmetric channel: application in crude oil refinement. J. Magn. Magn. Mater.,378, 463 - 468.*

13. *Job, VICTOR M., and S. Rao Gunakala. "Unsteady MHD Free Convection Couette Flow between Two Vertical Permeable Plates in the Presence of Thermal Radiation Using Galerkin's Finite Element Method." International Journal of Mechanical Engineering 2.5 (2013): 99-110.*
14. *Mahmoud, S.R., Afifi, N.A., & Al-Isede, H.M. (2011). Effect of porous media and magneticfield on peristaltic transport of a Jeffrey fluid in an asymmetric channel.International Journal of Mathematical Analysis, 5, 1025-1034.*
15. *N.S. Akbar. (2015). Heat transfer and carbon nano tubes analysis for the peristaltic flow in adiverging tube. Meccanica, 50, 39 - 47.*
16. *Nadeem, S., & Safia Akram. (2010). Heat transfer in a peristaltic flow of MHD fluid with partial slip. Communications in Nonlinear Science and Numerical Simulation, 15, 312 - 321.*
17. *Vajravelu, K., Radhakrishnamacharya, G., & Radhakrishnamurty, V. (2007). Peristaltic flow andheat transfer in a vertical porous annulus with long wave approximation. Int. J.NonLinear Mech.,42, 754 - 759.*
18. *Srinivas, S., &Kothandapani, M. (2008). Peristaltic transport in an asymmetric channel with heat transfer – A note. Int. Commun. Heat Mass Trans.,35, 514 - 522.*
19. *Akbar, N.S., & Nadeem, S.(2011). Simulation of heat transfer on the peristaltic flow of aJeffrey-six constant fluid in a diverging tube.International Communications in Heat andMass Transfer, 38, 154-150.*
20. *Ravikumar, S., & Abzal, S.K. (2017). Combined influence of hall currents and joule heating on hemodynamic peristaltic flow with porous medium through a vertical tapered asymmetric channel with radiation. Frontiers in Heat and Mass Transfer, 9-19, 1-9.*
21. *Ravikumar, S. (2018). Study of hall current, radiation and velocity slip on hydromagnetic physiological hemodynamic fluid with porous medium through joule heating and mass transfer in presence of chemical reaction,International Journal of Heat and Technology, 36(2), 422-432.*
22. *Eldabe, N. T. M., El-Sayed, M. F., Ghaly, A. Y., &Sayed, H. M. (2008). Mixed convective heat and mass transfer in a non-Newtonian fluid at a peristaltic surface with temperature- dependent viscosity. Archive of Applied Mechanics,78, 599 - 624.*
23. *Ogulu, T. A. (2006).Effect of heat generation on low Reynolds number fluid and mass transport in a single lymphatic blood vessel with uniform magnetic field. International Communications in Heatand Mass Transfer, 33, 790-799.*
24. *Nadeem, S., & Noreen Sher Akbar (2012). Effects of heat and mass transfer peristaltic flow of Williamson fluid in a vertical annulus. Meccanica, 47, 141 - 151.*